Physics 6C Fake Midterm

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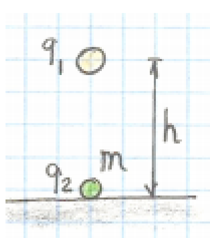
1 Problem 1

A small plastic sphere is charged to $-1\mu C$. It is held at $h = 1cm$ above a small glass bead at rest on a table. The bead has a charge of $+1\mu C$. What is the minimum mass of the bead so that it does not get pulled up to the plastic bead? Assume that the glass bead and the plastic sphere are point charges.



1.1 Solution to Problem 1

First we will want to draw a nice free body diagram of the glass bead. We will have the force of gravity acting downward on the glass bead and an electrostatic force upward due to the charges on both beads. Our job is to determine the minimum mass of the glass bead so that it does not get pulled upward. Or in other words when is the electrostatic force greater than the gravitational force. When they are equal is when the glass bead mass is at the limit for not moving up.

$$\begin{aligned}
\sum F =& 0 \\
F_e + F_g =& 0 \\
k\frac{q_1 q_2}{r^2} - mg =& 0 \\
k\frac{q_1 q_2}{r^2} =& mg \\
k\frac{q_1 q_2}{gr^2} =& m \longrightarrow m = 9 \cdot 10^9 \tfrac{Nm^2}{C^2} \frac{(1 \cdot 10^{-6} C)^2}{9.8\frac{m}{s^2}(0.01m)^2} \approx 9.2kg
\end{aligned}$$

2 Problem 2

An electron is placed into a uniform electric field, $E = 100,000 N/C$ with the initial velocity of $v_0 = 10^3 m/s$ **parallel** to the field. (a) What is the acceleration of the electron? (b) Does it eventually come to rest? If it does, how long does it take? If not, explain why not. (c) If the electric field were now perpendicular to the velocity of the electron (say the electron is traveling in the $+x$ direction and the electric field is in the $+y$ direction), how much does the electron get reflected after being inside field for $t = 2s$.

2.1 Solution to Problem 2 Part A

The acceleration of the electron can be found by first finding the force, then equating it to its mass times acceleration (Newtons second law!). The force is

$$\begin{aligned}
F =& qE \\
F =& e^- E
\end{aligned}$$

Now just invoke the second law to relate force to mass and acceleration

$$\begin{aligned}
ma =& e^- E \\
a =& \frac{e^- E}{m} \\
a =& \frac{-1.6 \cdot 10^{-19} C 100,000 N/C}{9.11 \cdot 10^{-31} kg} \approx -1.76 \cdot 10^{16} m/s^2
\end{aligned}$$

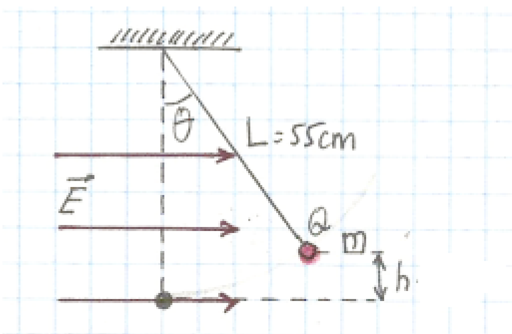
2.2 Solution to Problem 2 Part B

Since the electron experiences negative acceleration it will eventually come to rest and began to accelerate in the opposite direction as it was originally traveling. We could have also figured that out from the original setup since negative charges will always experience force in the opposite direction of the electric field. The time it takes it to stop can be determined from our kinematics knowledge (if you can remember that long ago! I sure as heck can't.) We will use the fact that when it stops it will have zero velocity (duh) and that it experiences constant acceleration.

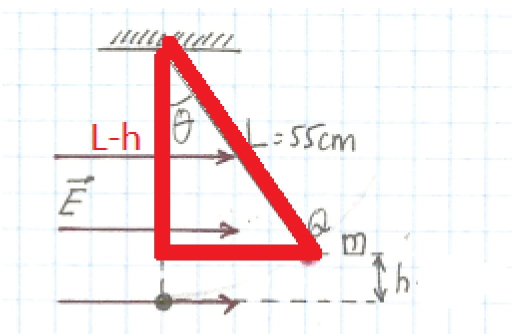
$$v = v_0 + at$$
$$-v_0 = at$$
$$\frac{-v_0}{a} = t \longrightarrow t = \frac{-10^3 \frac{m}{s}}{-1.76 \cdot 10^{16} m/s^2} \approx 5.7 \cdot 10^{-14} s$$

3 Problem 3

A point charge with the mass $m = 1g$ at the end of an insulating cord of length $55cm$ is observed to be in equilibrium in a uniform horizontal electric field of E, when the pendulums bob has $10\mu C$ of charge on it. If the height the bob rises is $h = 11cm$, find the magnitude of the electric field.



3.1 Solution to Problem 3



First we note that the system is in equilibrium. This means that the sum of the forces in both the x and the y directions are equal to zero. We will use this fact (and some trigonometry) to obtain the height the bob gets displaced.

$$\sum F_x = 0 \qquad \sum F_y = 0$$
$$F_e - T_x = 0 \qquad F_g - T_y = 0$$
$$qE = T sin\theta \qquad mg = T cos\theta$$

Now we have two equations and three unknown's. We need to determine what θ is. We will draw a right triangle on our diagram and use our knowledge trig to solve for theta.

$$cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - \frac{11cm}{55cm} = 0.8 \longrightarrow \theta = cos^{-1}(0.8) = 36.9^{\circ}$$

Now we just have to solve for E since we have as many equations as we do unknowns. We do not care about the tension T so we will solve for tension in one equation ($T = \frac{mg}{cos\theta}$) and plug into the other.

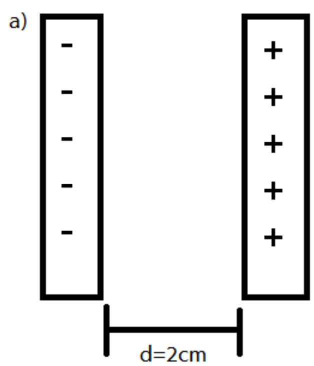
$$qE = Tsin\theta = \left(\frac{mg}{cos\theta}\right) cos\theta = mgtan\theta \longrightarrow E = \frac{mgtan\theta}{q}$$

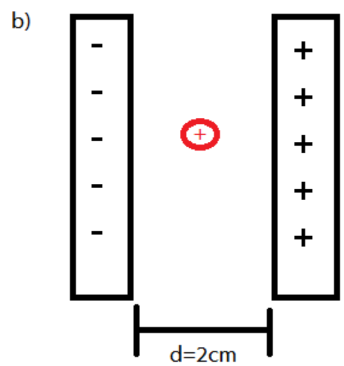
And plugging in the numbers we get

$$E \approx 75kN$$

4 Problem 4

We have parallel plate capacitor where both plates carry a surface charge density of magnitude of $\sigma = 1\mu C/m^2$ and are separated by a distance $d = 2cm$. (a) What is the electric field inside the capacitor? Draw the electric field in the figure below. (b) If we placed a proton in the center of the capacitor and let it go, what would its velocity be when it hit the negatively charged plate? Draw the electric field and label the forces. Hint: Use energy conservation!

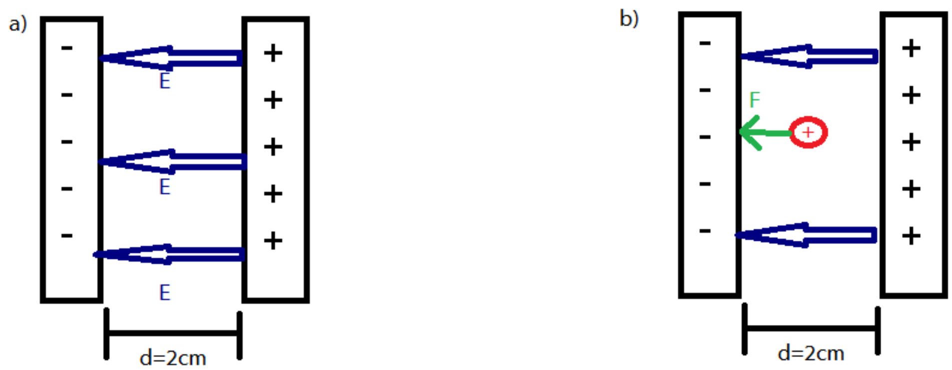




4.1 Solution to Problem 4 Part A

As we have determined in class the electric field due to one plate is $E_{plate} = 2\pi k\sigma$, where σ is the surface charge density. However, there are two plates so the electric field is just two times the electric field of one.

$$E_{cap} = 2E_{plate} = 4\pi k\sigma$$



4.2 Solution to Problem 4 Part B

Since the proton is placed halfway in the capacitor we know its potential energy is just the electric field times the displacement from the negative plate (see class lecture notes for more details)

$$U = qEd/2 = e^{+}(4\pi k\sigma d/2) = 1.6\cdot 10^{-19}C\cdot 4\pi\cdot 9\cdot 10^{9}\frac{Nm^2}{C^2}1\cdot 10^{-6}C/m^2\cdot 0.01m = 1.81\cdot 10^{-16}J$$

Now we just use energy conservation with initial kinetic energy and final potential energy equal to zero.

$$\begin{aligned} E_i =& E_f \\ U_i =& KE_f \\ e^{+}(4\pi k\sigma d/2) =& \frac{mv^2}{2} \longrightarrow v = \sqrt{\frac{e^{+}(4\pi k\sigma d)}{m}} = 465,600\frac{m}{s} \end{aligned}$$