Physics 5A Final Review Solutions

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1. A stone is dropped into the water from a tower 44.1m above the ground. Another stone is thrown vertically down 1.0 s after the first one is dropped. Both the stones strike the ground at the same time. What was the initial velocity of the second stone?

   Proof. The ball being dropped has motion described by $y = y_0 + v_0 t + \frac{1}{2}at^2$. Since the ball is being dropped its initial position and initial velocity is zero.

$$\begin{aligned} y =& y_0 + v_0 t_1 + \frac{1}{2}at^2 \\ y =& 0 + (0)t + \frac{1}{2}at_1^2 \\ \frac{2y}{a} =& t_1^2 \\ \sqrt{\frac{2y}{a}} =& t_1 \\ \sqrt{\frac{2 \cdot 44.1}{9.8}} =& t_1 = 3s \end{aligned}$$

   Now since the second ball is thrown down one second later we will subtract one second from t_1 to obtain time of travel for the second ball, $t_2 = 3s - 1s = 2s$. Now using the position equation for ball two we can determine v_0.

$$\begin{aligned} y =& y_0 + v_0 t_2 + \frac{1}{2}at_2^2 \\ y - \frac{1}{2}at_2^2 =& v_0 t_2 \\ \frac{y - \frac{1}{2}at_2^2}{t_2} =& v_0 \\ \frac{44.1 - \frac{1}{2}9.8\frac{m}{s^2}4s^2}{2s} =& v_0 = 12.25\frac{m}{s} \end{aligned}$$

   □

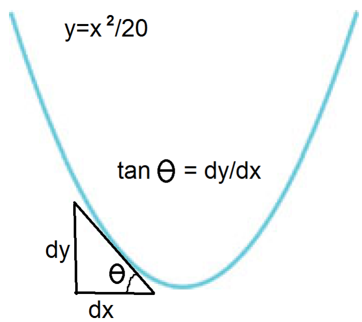
2. A uniform chain of length L lies on a table. If the coefficient of friction is μ, what is the maximum length of the part of the chain hanging over the table such that the chain does not slide?

   Proof. Let x be the length of the chain hanging over the table. The length of the chain resting on the table will be Lx. For equilibrium, gravitational force on the hanging part of the chain = frictional force on the part of the chain resting on the table. If M is the mass of the entire chain then

$$\begin{aligned}
\sum F &= ma = 0 \\
F_f - F_g &= 0 \\
F_f &= F_g \\
\frac{mgx}{L} &= \frac{m\left(L-x\right)g\mu}{L} \\
x &= \frac{\mu L}{\mu + 1}
\end{aligned}$$

Where I have left some algebra out because I am lazy. But it is trivial. □

3. A block is placed on a ramp of parabolic shape given by the equation $y = x^2/20$. If $\mu_s = 0.5$, what is the maximum height above the ground at which the block can be placed without slipping? If we suddenly turned off friction how fast would the block be moving at the lowest point of parabola.



Proof. Since $y = \frac{x^2}{20}$ then $\frac{dy}{dx} = \frac{x}{10} = tan\theta$. Now use a FBD to determine the equilibrium condition.

$$\begin{aligned}
\sum F &= ma = 0 \\
F_f - F_g &= 0 \\
F_f &= F_g \\
\mu mg cos\theta &= mg sin\theta \\
\mu &= tan\theta
\end{aligned}$$

Now from this we can find the x position

$$\frac{x}{10} = tan\theta \longrightarrow x = 10 tan\theta = 10\mu = 10 \times 0.5 = 5$$

Finally, we just plug into the equation of parabola to obtain

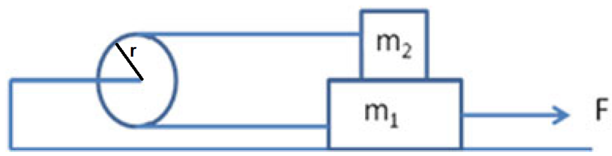
$$y(x=5) = \frac{5^2}{20} = 1.25m$$

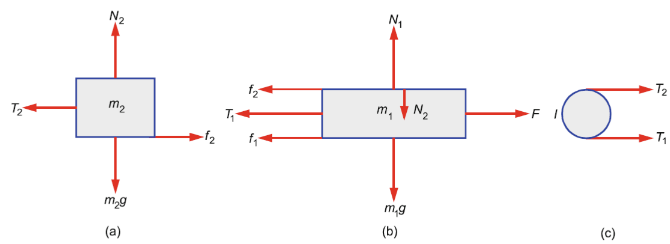
And the speed at the bottom is found by using energy conservation (gravitational potential gets converted entirely to kinetic)

$$mgh = \frac{mv^2}{2} \longrightarrow v = \sqrt{2gh}$$

□

4. Two blocks with masses m_1 and m_2 are attached by an unstretchable string around a frictionless pulley of radius r and moment of inertia I . Assume that there is no slipping of the string over the pulley and that the coefficient of kinetic friction between the two blocks and between the lower one and the floor is identical. If a horizontal force F is applied to m_1, calculate the acceleration of m_1





Free body diagrams for the two blocks and the pulley are shown below. The forces acting on m_2 are tension T_2 due to the string, gravity, frictional force f_2 due to the movement of m_1 and the normal force which m_1 exerts on it to prevent if from moving vertically. The forces on m_1 due to m_2 are equal and opposite to those of m_1 on m_2. By Newtons third law the tensions T_1 and T_2 in the thread are not equal as the pulley has mass. The equations of motion for m_1

$$\begin{aligned}\sum F_{1x} &= m_1 a \\ F - f_1 - f_2 - T_1 &= m_1 a\end{aligned}$$

and m_2

$$\begin{aligned}\sum F_{2x} &= m_2 a \\ T_2 - f_2 &= m_2 a\end{aligned}$$

and the pulley are

$$\begin{aligned}\sum \tau &= I\alpha \\ r(T_1 - T_2) &= I\frac{a}{r}\end{aligned}$$

Balancing the vertical forces

$$\begin{aligned}\sum F_{2y} &= 0 \\ N_2 - m_2 g &= 0 \\ N_2 &= m_2 g\end{aligned}$$

$$\begin{aligned}\sum F_{1y} &=0\\ -N_2 - m_2 g + N_1 &=0\\ N_1 &=N_2 + m_2 g\\ N_1 &=(m_1+m_2)g\end{aligned}$$

Frictional forces are

$$f_2 = \mu N_2 = \mu m_2 g \qquad\qquad f_1 = \mu N_1 = \mu(m_1+m_2)g$$

Now we can eliminate f_1, f_2, T_1, and T_2 by solving a system of equations (i.e. we have 5 equations and 5 unknowns). Adding the equations of motion for the horizontal directions we get

$$m_1 a + m_2 a = F - f_1 - f_2 - T_1 + T_2 - f_2$$

Then inserting our values for friction we obtain

$$m_1 a + m_2 a = F - \mu(m_1+m_2)g - 2\mu m_2 g - (T_1 - T_2)$$

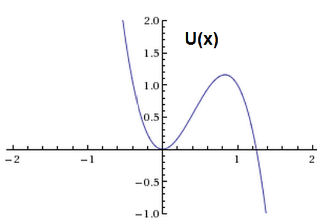
And then inserting our value for the difference of tensions (from torque equation) we obtain

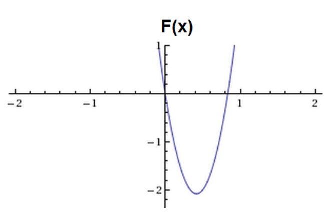
$$\begin{aligned} m_1 a + m_2 a &= F - \mu(m_1+m_2)g - 2\mu m_2 g - I\frac{a}{r^2}\\ I\frac{a}{r^2} + m_1 a + m_2 a &= F - \mu(m_1+m_2)g - 2\mu m_2 g\\ a(\tfrac{I}{r^2} + m_1 + m_2) &= F - \mu(m_1 + 3m_2)g\\ a &= \frac{F - \mu(m_1+3m_2)g}{\frac{I}{r^2} + m_1 + m_2}\end{aligned}$$

5. The potential energy of an object is given by

$$U(x) = 5x^2 - 4x^3$$

where U is in joules and x is in meters.
(i) Sketch the potential function and force function.
(ii) What is the force, $F(x)$, acting on the object?
(iii) Determine the positions where the object is in equilibrium and state whether they are stable or unstable.





Proof. We will use the definition of force to obtain the force function

$$F(x) = -\frac{dU}{dx} = -(10x - 12x^2) = 12x^2 - 10x$$

To find equilibrium we need to find the points where the force acting on the particle is zero. This can be found by finding the roots of the force function.

$$F(x) = 0 = 12x^2 - 10x = x(12x - 10) \longrightarrow x = 0, x = \frac{5}{6}$$

Now, to check if the positions of equilibrium are stable, we simply check whether or not potential function concave up, (not in equilibrium) or concave down(equilibrium). Or if force function is restoring or not. First we will check $x = 0$.

$$\frac{d^2U}{dx} = \frac{dF}{dx}\Big|_{x=0} = 24x - 10\Big|_{x=0} = -10$$

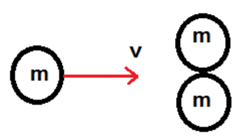
This is in equilibrium. The particle will tend to return to original position if displaced slightly. For $x = 5/6$ we see

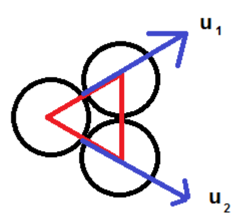
$$\frac{d^2U}{dx} = \frac{dF}{dx}\Big|_{x=5/6} = 24x - 10\Big|_{x=5/6} = 20 - 10 = 10$$

This position is unstable. If a small force displaces the particle the particle will accelerate away from equilibrium position.

□

6. Two billiard balls are resting on a smooth table, and just touching. A third identical ball moving along the table with velocity v perpendicular to their line of centres strikes both balls simultaneously. Find the velocities of the three balls immediately after impact, assuming that the collision is elastic.





Proof. From the figure above we see that the centers of the billiard balls make a equilateral triangle. The force imparted one the two stationary balls is perpendicular to the tangent plane of both contact points. Therefore, the direction of the two velocities of the originally stationary balls are 60° apart. This means they are traveling at 30° and -30° from the original direction of motion. Now we can use our conservation laws to determine the resulting velocities. We start with momentum conservation and noting that there is no net external force applied to the incoming ball in the y-direction, hence no final momentum in the y-direction.

$$\begin{aligned} &Y\!-\!Component \\ P_{iy} &= P_{fy} \\ 0 &= mu_1 sin30^\circ + mu_2 sin(-30^\circ) \\ u_1 &= u_2 \longrightarrow |u_1| = |u_2| = u \end{aligned}$$

$$\begin{aligned} &X\!-\!Component \\ P_{ix} &= P_{fx} \\ mv_0 &= mv + mucos30^\circ + mucos30^\circ \\ v_0 - v &= \sqrt{3}u \end{aligned}$$

Now we can write down energy conservation as our third equation for our three unknowns.

$$\begin{aligned} E_i =& E_f \\ \frac{mv_0^2}{2} =& \frac{mv^2}{2} + \frac{mu_1^2}{2} + \frac{mu_2^2}{2} \\ v_0^2 =& u_1^2 + u_2^2 + v^2 \\ v_0^2 =& u^2 + u^2 + v^2 \end{aligned}$$

Now we have 3 equations and 3 unknowns. First we will manipulate the energy equation to obtain a difference of squares, and plug in a difference from the momentum equations.

$$\begin{aligned} v_0^2 - v^2 =& 2u^2 \\ (v_0 - v)(v_0 + v) =& 2u^2 \\ (v_0 + v)\sqrt{3}u =& 2u^2 \\ (v_0 + v)\sqrt{3} =& 2u \end{aligned}$$

Now plug this back into the x-component momentum equation to obtain

$$\begin{aligned} v_0 - v =& \sqrt{3}u \\ v_0 - v =& \sqrt{3}(v_0 + v)\frac{\sqrt{3}}{2} \\ v_0 - v =& \frac{3}{2}v_0 + \frac{3}{2}v \\ -\frac{5v}{2} =& \frac{v_0}{2} \\ v =& -\frac{v_0}{5} \end{aligned}$$

Taking this value and plugging into the previous equation we obtain

$$\begin{aligned} (v_0 + v)\sqrt{3} =& 2u \\ (v_0 + \frac{-v_0}{5})\sqrt{3} =& 2u \\ \frac{4\sqrt{3}v_0}{5} =& 2u \\ \frac{2\sqrt{3}v_0}{5} =& u \end{aligned}$$

Now that we know the magnitude of u we can find its vector components by multiplying it by a the sine or cosine of 30 degrees.

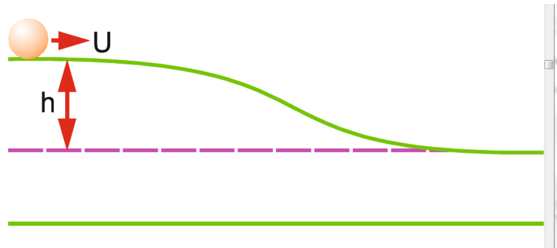
$$u_x = \frac{2\sqrt{3}v_0}{5} cos(\pm 30°) = \frac{2\sqrt{3}v_0}{5}\frac{\sqrt{3}}{2} = \frac{3}{5}v_0 \qquad u_y = \frac{2\sqrt{3}v_0}{5} sin(\pm 30°) = \pm\frac{2\sqrt{3}v_0}{5}\frac{1}{2} = \pm\frac{\sqrt{3}}{5}v_0$$

Applying these values to the corresponding particles we find that the resultant velocities are

$$v = \left(-\frac{1}{5}, 0\right) v_0 \qquad u_1 = \left(\frac{\sqrt{3}}{5}, \frac{3}{5}\right) v_0 \qquad u_2 = \left(\frac{\sqrt{3}}{5}, -\frac{3}{5}\right) v_0 \tag{1}$$

□

7. A disc rolls without slipping along a horizontal surface with velocity u. The disc then encounters a smooth drop of height h, after which it continues to move with velocity v. At all times the disc remains in a vertical plane. Determine the value of v in terms of g, h and u.



Proof. We will use conservation of energy to determine the velocity v after the disc has rolled down the slope it will convert of the potential energy into rotational kinetic energy and translational kinetic energy. So there is no external forces we will only need to consider conservation of energy without work done on system. We will use the fact that the disc is not slipping hence $\omega_i r = u$ and $v = \omega_f r$

$$\begin{aligned}
E_i =& E_f \\
KE_{Ti} + KE_{Ri} + U_{Gi} =& KE_{Rf} + KE_{Tf} + U_{Gf} \\
\frac{mu^2}{2} + \frac{I\omega_i^2}{2} + mgh =& \frac{mu^2}{2} + \frac{I\omega_i^2}{2} + mg \cdot 0 \\
\frac{mu^2}{2} + \frac{\frac{mr^2}{2}\frac{u^2}{r^2}}{2} + mgh =& \frac{mv^2}{2} + \frac{\frac{mr^2}{2}\frac{v^2}{r^2}}{2} \\
u^2 + \frac{u^2}{2} + 2gh =& v^2 + \frac{v^2}{2} \\
v^2 + \frac{v^2}{4} =& u^2 + \frac{u^2}{2} + 2gh \\
v^2(1 + \tfrac{1}{2}) =& \frac{3u^2}{2} + 2gh \\
v^2 =& u^2 + \frac{4gh}{3} \\
v =& \pm\sqrt{u^2 + \frac{4gh}{3}}
\end{aligned}$$

□

8. Consider a point mass m with momentum p rotating at a distance r about an axis. Starting from the definition of the angular momentum $L \equiv r \times p$ of this point mass, show that

$$\frac{dL}{dt} = \tau$$

where τ is the torque.

Proof. First we will use product rule to find the derivative of L

$$\frac{dL}{dt} = \frac{d}{dt} r \times p = r \times \frac{dp}{dt} + \frac{dr}{dt} \times p$$

Now we note that $v(= \frac{dr}{dt})$ is parallel to p hence $v \times p = 0$. Also, we note that by definition $F = \frac{dp}{dt}$

$$r \times \frac{dp}{dt} + \frac{dr}{dt} \times p = r \times \frac{dp}{dt} + 0 = r \times F = \tau$$

Where $r \times F = \tau$ by definition. $\square$

9. A uniform rod of length l and mass M rests on a frictionless horizontal surface. The rod pivots about a fixed frictionless axis at one end. The rod is initially at rest. A bullet of mass m travelling parallel to the horizontal surface and perpendicular to the rod with speed v strikes the rod at its centre and becomes embedded in it. Using the result of the previous question, show that the angular momentum of the rod after the collision is given by

$$|L| = \frac{1}{2}lv$$

(i) Is $\vec{L} = (l/2)m\vec{v}$ also correct?
(ii) What is the final angular speed of the rod?
(iii) Assuming $M = 5m$, what is the ratio of the kinetic energy of the system after the collision to the kinetic energy of the bullet before the collision?

Proof. (i) This expression for the VECTOR L is not correct because the vector v is perpendicular to the vector L. (ii) Using conservation of angular momentum we find that

$$\begin{aligned}
L_i =& L_f \\
I_m \omega_i =& I_M \omega_f + I_m \omega_f \\
mr^2 \frac{v}{r} =& \frac{1}{3} ml^2 \omega_f + mr^2 \omega_f \\
mv\frac{l}{2} =& \frac{1}{3} Ml^2 \omega_f + m\frac{l^2}{4}\omega_f \\
mv\frac{l}{2} =& \left(\frac{1}{3}Ml^2 + m\frac{l^2}{4}\right)\omega_f \\
\omega_f =& \frac{mv\frac{l}{2}}{\left(\frac{1}{3}Ml^2 + m\frac{l^2}{4}\right)} \\
\omega_f =& \frac{6mv}{(4M + 3m)\, l}
\end{aligned}$$

(iii) The final energy just after collision is entirely rotational, and before collision is entirely kinetic. Hence initial energy is just

$$KE_i = \frac{1}{2}mv^2$$

and the final rotational kinetic energy is just

$$KE_f = \frac{1}{2}m\frac{l^2}{4}\omega_f^2 + \frac{1}{3}m\frac{l^2}{4}\omega_f^2$$

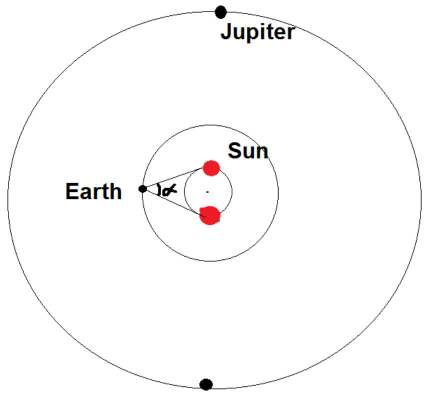
Where I will omit the simple algebra to obtain KE_f as

$$KE_f = \frac{3m^2v^2}{2\,(4M + 3m)} = \frac{3mv^2}{46}$$

Now the ratio of initial energy to final energy is

$$\frac{KE_f}{KE_i} = \frac{\frac{3mv^2}{46}}{\frac{mv^2}{2}} = \frac{3}{23}$$

$\square$



10. Where is the center of mass of the SunJupiter system? (The mass ratio is $M_S/M_J = 1047$. Through what angle does the Suns position as seen from the Earth oscillate because of the gravitational attraction of Jupiter? The distance from earth to the sun is $1AU = 1.5 \times 10^{11}m$, and the jupiter sun sun distance is $R_{j-s} = 5.2AU$.

    Proof. To determine the center of mass we will need to first define a origin, or reference point. The reference point of the sun is convenient because then the suns position is zero and jupiters position is just the radius of jupiters orbit. The center of mass is defined as $\vec{R} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$. Using this definition we find that the center of mass (relative to sun) is just

$$\vec{R} = \frac{m_s \cdot 0 + m_j r_{js}}{m_s + m_j} = \frac{m_j r_{js}}{m_j \left(1 + \frac{m_s}{m_j}\right)} = \frac{r_{js}}{\left(1 + \frac{m_s}{m_j}\right)} = \frac{5.2AU}{1 + 1047} = 0.005AU \approx 7.5 \times 10^5 km \tag{2}$$

    The angle that the sun shifts is given by the arc length equation for small arc, or sine function for small angles, where the arc length is the radius of the orbit than the sun makes around the sun-jupiter center of mass, which is just the center of mass position. The radius was found in part A to be $R = 7.5 \times 10^5 km$. Hence the angle is

$$\theta = \frac{s}{r} = \frac{R}{r_{js}} = \frac{7.5 \times 10^5 km}{1.5 \times 10^8 km} = 0.005\ radians = 0.286° \tag{3}$$

    □

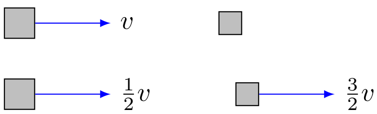
11. A rocket of mass 1000Mg (= 1 mega gram=$10^6 kg$) has an upward acceleration equal to 0.5g . How many kilograms of fuel must be ejected per second at a relative speed of $2000\frac{m}{s}$ to produce the desired acceleration?

    Proof. Since we know its acceleration we can sum the forces on the rocket to determine the rate of mass expelled.

$$\begin{aligned}
\sum F &= ma \\
-mg + v_r \frac{dm}{dt} &= m(0.5g) \\
v_r \frac{dm}{dt} &= \frac{3mg}{2} \\
\frac{dm}{dt} &= \frac{3mg}{2v_r} \\
\frac{dm}{dt} &= \frac{3(1.5 \times 10^g kg \cdot 9.8\frac{m}{s^2})}{2000\frac{m}{s}} = 7350\frac{kg}{s}
\end{aligned}$$

    □

12. An object A moving with velocity v collides with a stationary object B. After the collision, A is moving with velocity $\frac{1}{2}v$ and B is moving with velocity $\frac{3}{2}v$. (a) Find the ratio of masses. (b) If the masses stuck together what velocity would they have after colliding?



Part A

We first note that the this is a conservation of momentum problem. Since it is also an elastic collision we can say energy is also conserved. We write down the equations for conservation of momentum since it is all we need to solve.

$$\begin{aligned} P_i &= P_f \\ m_A v &= \frac{m_A v}{2} + \frac{3m_B v}{2} \\ m_A v - \frac{m_A v}{2} &= \frac{3m_B v}{2} \\ \frac{m_A v}{2} &= \frac{3m_B v}{2} \\ \frac{m_A}{m_B} &= 3 \end{aligned} \tag{4}$$

Part B

We note that this is a non-elastic collision hence energy is not necessarily conserved. We write down conservation of momentum equations.

$$\begin{aligned} P_i &= P_f \\ m_A v &= (m_A + m_B)\, v' \\ \frac{m_A v}{(m_A + m_B)} &= v' \\ \frac{m_A v}{m_B} \frac{1}{\left(\frac{m_A}{m_B} + 1\right)} &= v' \\ 3\frac{v}{3+1} &= v' \\ \frac{3v}{4} &= v' \end{aligned} \tag{5}$$

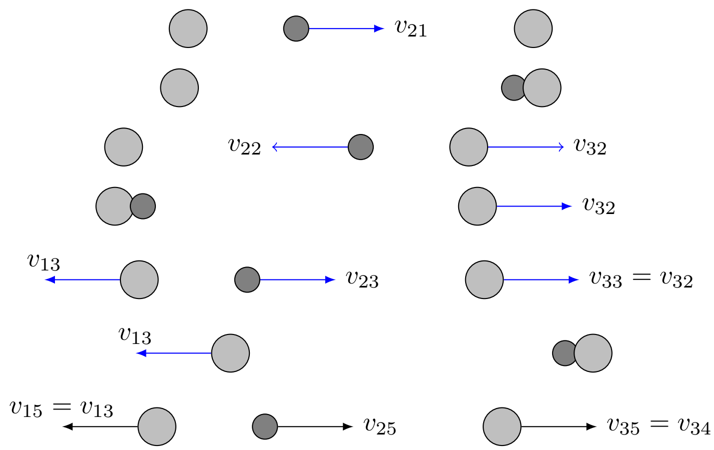
1 Challenge Problem!

13. Three perfectly elastic bodies of masses 5 kg, $1kg$, $5kg$ are arranged in that order on a straight line, and are free to move along it. Initially, the middle one is moving with velocity $27\frac{m}{s}$, and the others are at rest. Find how many collisions take place in the subsequent motion, and verify that the final value of the kinetic energy is equal to the initial value.

Proof. **1.0.1 Initial energy**

There is only kinetic energy in the initial situation given by

$$E = T_i = \frac{m_2 v_{21}^2}{2} = 364.5J$$



1.0.2 First Collision

To determine the resulting velocities v_{22} and v_{32}. Since it is an elastic collision we can say energy is also conserved. We first write down the equations for conservation of momentum and conservation of energy.

$$\begin{aligned} P_{1i} &= P_{1f} \\ m_2 v_{21} &= -m_2 v_{22} + m_3 v_{32} \\ m_2 v_{21} + m_2 v_{22} &= m_3 v_{32} \\ m_2 \left(v_{21} + v_{22}\right) &= m_3 v_{32} \end{aligned} \tag{6}$$

$$\begin{aligned} E_{1i} &= E_{1f} \\ \frac{1}{2} m_2 v_{21}^2 &= \frac{1}{2} m_2 v_{22}^2 + \frac{1}{2} m_3 v_{32}^2 \\ \frac{1}{2} m_2 v_{21}^2 - \frac{1}{2} m_2 v_{22}^2 &= \frac{1}{2} m_3 v_{32}^2 \\ m_2 \left(v_{21}^2 - v_{22}^2\right) &= \frac{1}{2} m_3 v_{32}^2 \\ m_2 \left(v_{21} - v_{22}\right)\left(v_{21} + v_{22}\right) &= m_3 v_{32}^2 \end{aligned} \tag{7}$$

1.0.3 Second Collision

Now we divide the energy equation by the momentum equation to obtain $v_{21} - v_{22} = v_{32}$. Plugging this back into the momentum equation we get

$$\begin{aligned} m_2 \left(v_{21} + v_{22}\right) &= m_3 v_{32} \\ m_2 v_{21} + m_2 v_{22} &= m_3 v_{21} - m_3 v_{22} \\ -m_3 v_{22} - m_2 v_{22} &= m_2 v_{21} - m_3 v_{21} \\ v_{22} &= \frac{\left(m_3 - m_2\right) v_{21}}{\left(m_3 + m_2\right)} \\ v_{22} = -\frac{2}{3} v_{21} &= -\frac{2}{3} 27 \tfrac{m}{s} = -18 \tfrac{m}{s} \end{aligned}$$

We can then find v_{32} by $v_{32} = v_{21} - v_{22} = 9\frac{m}{s}$. The second ball then begins traveling towards the first ball. We have the same situation with a ball traveling with initial velocity then striking a stationary ball. We can use the same equations as derived previously to find the resulting velocities. The equation for velocities of this new situation is $v_{13} = v_{22} + v_{23}$. Plugging back into the momentum equation we obtain

$$v_{23} = \frac{\left(m_1 - m_2\right) v_{22}}{\left(m_1 + m_2\right)} = \frac{2}{3} v_{22} = 12 \tfrac{m}{s}$$

After the collision ball 1 will have velocity $v_{13} = v_{22} + v_{23} = 12\frac{m}{s} - 18\frac{m}{s} = -6\frac{m}{s}$. Ball 2 will hit ball 3 again because ball 2 has a larger velocity than ball 3 ($v_{23} > v_{32}$).

1.0.4 Third Collision

Since now we have a collision between two moving particles we must derive an equation for the resulting velocities using the same technique.

$$\begin{aligned} P_{2i} &= P_{2f} \\ m_2 v_{23} + m_3 v_{33} &= m_2 v_{24} + m_3 v_{34} \\ m_2 v_{23} - m_2 v_{24} &= -m_3 v_{33} + m_3 v_{34} \\ m_2 \left(v_{23} - v_{24}\right) &= m_3 \left(v_{34} - v_{33}\right) \end{aligned} \quad (8)$$

$$\begin{aligned} E_{2i} &= E_{2f} \\ \frac{1}{2} m_2 v_{23}^2 + \frac{1}{2} m_3 v_{33}^2 &= \frac{1}{2} m_2 v_{24}^2 + \frac{1}{2} m_3 v_{34}^2 \\ \frac{1}{2} m_2 v_{23}^2 - \frac{1}{2} m_2 v_{24}^2 &= \frac{1}{2} m_3 v_{34}^2 - \frac{1}{2} m_3 v_{33}^2 \\ m_2 \left(v_{23}^2 - v_{24}^2\right) &= m_3 \left(v_{34}^2 - v_{33}^2\right) \\ m_2 \left(v_{23} - v_{24}\right)\left(v_{23} + v_{24}\right) &= m_3 \left(v_{34} - v_{33}\right)\left(v_{34} + v_{33}\right) \end{aligned} \quad (9)$$

Dividing these equations and then plugging back into momentum equation we obtain

$$\begin{aligned} \frac{m_2 \left(v_{23} - v_{24}\right)\left(v_{23} + v_{24}\right)}{m_2 \left(v_{23} - v_{24}\right)} &= \frac{m_3 \left(v_{34} - v_{33}\right)\left(v_{34} + v_{33}\right)}{m_3 \left(v_{34} - v_{33}\right)} \\ v_{23} + v_{24} &= v_{34} + v_{33} \\ v_{34} &= v_{23} + v_{24} - v_{33} \end{aligned}$$

$$\begin{aligned} m_2 v_{23} + m_2 v_{24} &= m_3 v_{33} - m_3 v_{34} \\ m_2 v_{23} + m_2 v_{24} &= m_3 v_{33} - m_3 \left(v_{23} + v_{24} - v_{33}\right) \\ m_2 v_{23} + m_2 v_{24} &= m_3 v_{33} - m_3 v_{23} - m_3 v_{24} + m_3 v_{33} \\ m_3 v_{24} + m_2 v_{24} &= m_3 v_{33} - m_2 v_{23} - m_3 v_{24} + m_3 v_{33} \\ \left(m_2 + m_3\right) v_{24} &= m_3 v_{33} - m_2 v_{23} - m_3 v_{23} + m_3 v_{33} \\ v_{24} &= \frac{m_3 v_{33} - m_2 v_{23} - m_3 v_{23} + m_3 v_{33}}{\left(m_3 + m_2\right)} \\ v_{24} &= \frac{2 m_3 v_{33} - \left(m_3 + m_2\right) v_{23}}{\left(m_3 + m_2\right)} \end{aligned}$$

Plugging the numbers in we obtain $v_{24} = 7\frac{m}{s}$ and $v_{34} = v_{23} + v_{24} - v_{33} = 9\frac{m}{s} + 7\frac{m}{s} - 6\frac{m}{s} = 10\frac{m}{s}$

1.0.5 Final Energy and Number of Collisions

The final energy is given by the sum of the individual kinetic energies.

$$T_f = \frac{m_1 v_{14}^2}{2} + \frac{m_2 v_{24}^2}{2} + \frac{m_3 v_{34}^2}{2} = \frac{5kg \cdot 6\frac{m}{s}^2}{2} + \frac{1kg \cdot 7\frac{m}{s}^2}{2} + \frac{5kg \cdot 10\frac{m}{s}^2}{2} = 364.5J$$

We also note there was only 3 collisions. □