Physics 5A Review Problems

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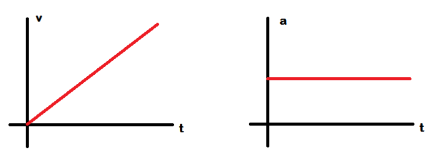
1 Short Response

1. Does odometer measure distance or displacement? **Distance**

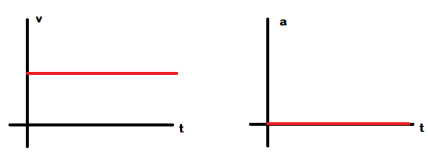
2. Does speedometer measure speed of velocity? **speed**

3. Graph and describe the acceleration and velocity of each function of position.

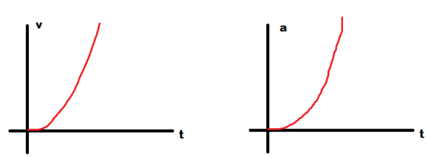
   A. $x = t^2$



   B. $x = 2t$



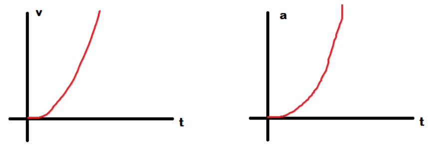
   C. $x = e^t$



   Please note that these graphs should be equivalent.

4. A plane accelerates at $5\frac{m}{s^2}$ and needs to have a velocity of $50\frac{m}{s}$ to take off. At what time does the plane take off at? Assume at $t = 0$, $v = 0$ and $x = 0$. How far does the plane travel in that time? A. 5 seconds , 50 meters B. 50 seconds, 250 meters **C. 10 seconds, 500 meters** D. 1 second, 100 meters

5. If $a(t) = a_0 t$ draw the graph of velocity versus time, and position versus time. Where a_0 is some positive constant.



6. I launch a projectile into the air at 45° from the horizontal. Remember that the projectile gets acted upon by gravity. What is the equation of position for the x-direction. $x = x_0 + vt$

7. A ball thrown downward and a ball dropped which one has the greatest acceleration? **They are equal**

8. A ball thrown upward and a ball dropped which one has the greatest acceleration? **They are equal**

9. A ball thrown upward and a ball thrown vertically downward, with the same initial velocity. Which one has the greatest velocity when they hit? **They are equal**

10. Does the force of friction always opposes the direction of motion? **Technically no. But in this context the answer is yes.**

11. If your have a constant speed can you be accelerating? If yes give an example. If no explain. **Yes you can. That's what uniform circular motion is!**
    A. $30\frac{m}{s^2}$, 20580 N
    B. $450\frac{m}{s^2}$, 308700 N
    C. $900\frac{m}{s^2}$,617400 N
    D. $300\frac{m}{s^2}$, 205800 N

12. You are driving and want to stop as quickly as possible, you should:
    A. slam on the brakes as hard as you can and skid to stop
    B. smash into the nearest object
    C. firmly press the brakes, with out skidding

13. Which of the following statements are true, in regards to the apex (highest point) of a trajectory.
    A. The projectiles vertical acceleration is zero.
    B. The projectiles horizontal acceleration is not zero.
    C. The projectiles vertical velocity is zero.
    D. The projectiles horizontal velocity is zero.

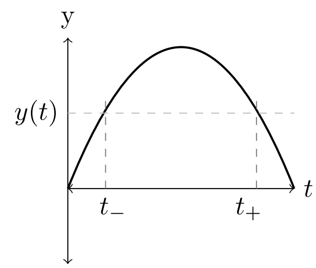
14. $8.000 \times 1.000 \times 3.0 + 2 =$
    A. 26.000
    B. 26.00
    C. 26.0
    D. 26

2 Free response problems

1. Find the times where the y-positions are equal in a trajectory. We start with our equation for position.

$$
\begin{aligned}
y(t) =& y_0 + v_0 t - \frac{gt^2}{2} \\
\frac{gt^2}{2} - v_0 t + (y(t) - y_0) =& 0 \\
t^2 - \frac{2v_0 t}{g} + (y(t) - y_0) =& 0
\end{aligned}
$$

Now we have a quadratic where there are two roots. This physically means that there are two times at which the y-position is equal to any arbitrary height, $y(t)$.

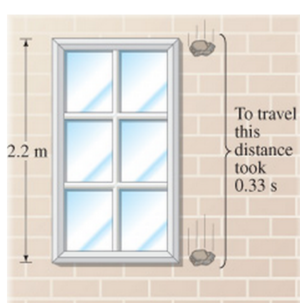


$$
\begin{aligned}
t^2 - \frac{2v_0}{g} t + \frac{2}{g}(y_2 - y_0) =& 0 \\
t =& \frac{\frac{2v_0}{g} \pm \sqrt{\left(\frac{2v_0}{g}\right)^2 - 4(1)\frac{2}{g}(y_2 - y_0)}}{2(1)} \\
t =& \frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 - (1)\frac{2}{g}(y_2 - y_0)}
\end{aligned}
$$

Which gives the times which the projectile is at some arbitrary height in its trajectory as

$$
t_+ = \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 - \frac{2}{g}(y_2 - y_0)} \qquad t_- = \frac{v_0}{g} - \sqrt{\left(\frac{v_0}{g}\right)^2 - \frac{2}{g}(y_2 - y_0)}
$$

2. A falling stone takes 0.33 s to travel past a window 2.2 m tall. Your friend throw it straight down from a 0.5m above the top of the window. What velocity did your friend throw it at?



We have to break this problem into parts. First is to find the velocity at the top of the window. Then we can use the initial velocity (when we first see the rock) at the top of the window as the final velocity of rock being dropped from rest.

2.0.1 First Part

So we know the time, the distance and the acceleration so we would use

$$z(t) = z_0 + v_0 t + \frac{gt^2}{2} \longrightarrow \Delta z = v_0 t + \frac{gt^2}{2}$$

Where Δz is the height of the window. Now we just solve for v_0

$$\begin{aligned} \Delta z =& v_0 t + \frac{gt^2}{2} \\ \Delta z - \frac{gt^2}{2} =& v_0 t \\ \frac{\Delta z - \frac{gt^2}{2}}{t} =& v_0 \end{aligned}$$

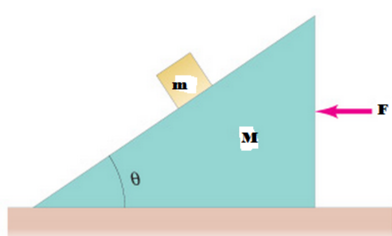
Now that we have solved for the velocity at the top of the window we can find at which height (above the window) a rock would have to be dropped to obtain that velocity. We have defined Δz as the height above the window. We want an equation that is independent of time and that has acceleration ,position and velocity. That equation is

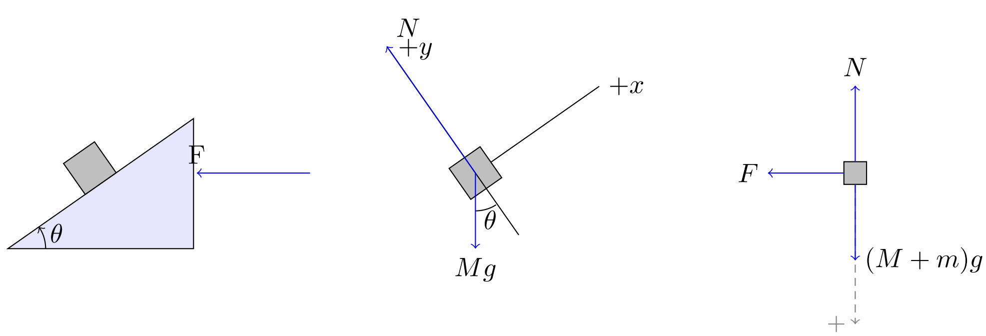
$$v_{final}^2 - v_{initial}^2 = 2a\Delta z$$

Where in this equation $v_{initial}$ is what we are looking for and $v_{final} = v_0$ from the previous part. Solving for $v_{initial}$ we find the height.

$$v_{initial}^2 = v_{final}^2 + 2gh = \left(\frac{\Delta z - \frac{gt^2}{2}}{t} \right)^2 + 2g\Delta z \tag{1}$$

3. There is a wedge of mass M, and slope *theta*, that is free to move on a table. There is small mass m that sits on the slope of the wedge. What force, F, on the big block is needed to keep the little block from sliding down the wedge?





For this situation we want to use a coordinate axis that is not tilted. We want y-axis vertical and x-axis horizontal. For the mass to not accelerate we want a normal force that exactly counteracts the force of gravity on the small block. Hence, there will some acceleration in the x-direction and no acceleration in the y-direction. Also the free body diagram of the wedge includes the mass of m, since the force is "sees" an object of mass $M + m$ not just M. Now all we do is use our free body diagrams and Newtons second Law. For the wedge we have

$$\begin{aligned} \sum F_x &= (M+m)a \\ F &= (M+m)a \end{aligned} \qquad \begin{aligned} \sum F_y &= (M+m)a = 0 \\ N_M - (M+m)g &= 0 \\ N_M - (M+m)g &= 0 \end{aligned}$$

Now for the small block of mass m we have

$$\begin{aligned} \sum F_x &= ma \\ N sin\theta &= ma \end{aligned} \qquad \begin{aligned} \sum F_y &= ma = 0 \\ N cos\theta - mg &= 0 \\ N &= \frac{mg}{cos\theta} \end{aligned}$$

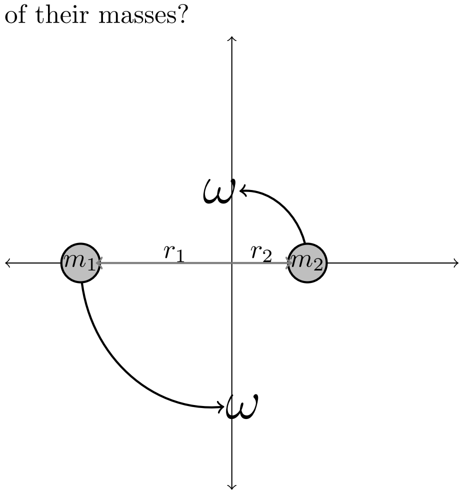
Plugging the N found from the y-direction summation of forces into the x-direction summation of forces equation we obtain

$$\begin{aligned} N sin\theta &= ma \\ \frac{mg}{cos\theta} sin\theta &= ma \\ g tan\theta &= a \end{aligned}$$

Finally, we plug this value of the acceleration into the summation of forces equation of the wedge (and block!) to obtain

$$\begin{aligned} F &= (M+m)a \\ F &= (M+m) g tan\theta \qquad (2) \end{aligned}$$

4. The two components of a double star are observed to move in circles of radii r_1 and r_2. What is the ratio of their masses?



We know that both stars exert equal and opposite forces of each other, $F_{12} = -F_{21}$. This is the only force that could cause angular acceleration. Both stars feel the same angular acceleration. The stars separation is given by $\Delta\vec{r} = \vec{r_1} - \vec{r_2}$ and the force on each star is given by

$$F_{12} = -F_{21} = G\frac{Mm}{\Delta\vec{r}}$$

We apply Newtons second law to obtain

$$F_{12} = ma_{c_1} = m\omega^2 r_1 \qquad\qquad F_{21} = Ma_{c_2} = M\omega^2 r_2$$

Finally we take the ratios of the forces an solve for the ratio of radii

$$\begin{aligned}\frac{F_{12}}{F_{21}} &= \frac{m_1 a_{c_1}}{m_2 a_{c_2}} \\ \frac{G\frac{m_1 m_2}{\Delta\vec{r}}}{G\frac{m_2 m_1}{\Delta\vec{r}}} &= \frac{m_1\omega^2 r_1}{m_2\omega^2 r_2} \\ 1 &= \frac{m_1 r_1}{m_2 r_2} \\ \frac{r_2}{r_1} &= \frac{m_1}{m_2}\end{aligned} \tag{3}$$

5. An aircraft is to fly to a destination $800km$ due north of its starting point. Its airspeed is $800\frac{km}{hr}$. The wind is from the east at a speed of $30\frac{m}{s}$. On what compass heading should the pilot fly? How long will the flight take? If the wind speed increases to $50\frac{m}{s}$, and the wind backs to the north-east, but no allowance is made for this change, how far from its destination will the aircraft be at its expected arrival time, and in what direction?

2.0.2 Part A

I will first convert the wind speed from ms^{-1} to kmh^{-1}.

$$30\frac{m}{s} \times \frac{3600sec}{1hr} \times \frac{1km}{1000m} = 108\frac{km}{hr}$$

Having the wind and airplane velocities in same units we will just use vector addition, with $v_p = \left(0\hat{\imath}, 800\hat{\jmath}\right)\frac{km}{hr}$ and $v_w = \left(-108\hat{\imath}, 0\hat{\jmath}\right)\frac{km}{hr}$ with a combined velocity of $v_t = \left(-108\hat{\imath}, 800\hat{\jmath}\right)\frac{km}{hr}$. To get the angle with which the plane should leave we must find the angle at which the planes speed will contribute $108\frac{km}{hr}$ to the east.

$$\begin{aligned}800\frac{km}{hr}\cdot sin\left(\theta\right) =& 108\frac{km}{hr} \\ \theta =& sin^{-1}\left(\frac{108\frac{km}{hr}}{800\frac{km}{hr}}\right) = 7.8^\circ\end{aligned} \tag{4}$$

This answer is quoted as 7.8° east of north.

2.0.3 Part B

Since the the destination is directly north we use trigonometry to determine the y-velocity at 7.6° north of west heading. The y-velocity is then just

$$v_y = 800\frac{km}{hr}cos\left(7.8^\circ\right) = 792\frac{km}{hr}$$

Hence, the time it takes to reach the destination is then just the distance divided by the y- velocity.

$$t = \frac{800km}{792\frac{km}{hr}} = 1.01hr = 60.6min \tag{5}$$

2.0.4 Part C

If the wind increases to $50\frac{m}{s}$ $(180\frac{km}{hr})$ from the north-east and no corrections are made the corresponding velocity vector of the wind would be $v_w = \left(-127\hat{i}, -127\hat{j}\right)\frac{km}{hr}$. Adding this velocity to the planes velocity vector we obtain

$$v_w = \left(108 - 127\hat{i}, 792 - 127\hat{j}\right)\frac{km}{hr} = \left(-19\hat{i}, 665\hat{j}\right)\frac{km}{hr}$$

.

Now by multiplying each component by the expected time we will get the position

$$\vec{r} = \left(-19\hat{i}, 665\hat{j}\right)\frac{km}{hr} \times 1.01hr = \left(-19.2\hat{i}, 672\hat{j}\right)$$

The distance from the destination is then just the vector subtraction of the planes position from destinations position $\vec{d} = \left(0 - (-19.2)\hat{i}, 800 - 772\hat{j}\right)$ and then use Pythagorean theorem to obtain

$$d = \sqrt{19.2^2 + 128^2} = 129.4km \approx 130km \tag{6}$$

With an angle of

$$\theta = tan^{-1}\left(\frac{-128}{19.2}\right) = 8.5°\text{W of S} \qquad 98.46° \tag{7}$$

3 Challenge Problem!

Find the equation for the trajectory of a projectile launched with velocity v at an angle α to the horizontal, assuming negligible atmospheric resistance. Given that the ground slopes at an angle β, show that the range of the projectile (measured horizontally) is

$$x = \frac{2v^2}{g}\frac{sin(\alpha - \beta)cos\alpha}{cos\beta}$$

At what angle should the projectile be launched to achieve the maximum range?

3.0.5 Part A

To solve we first write down equations for the x and z directions:

X-Direction

$$x = x_0 + v_{0x}t + \frac{at^2}{2}$$
$$x = v_0 t cos\alpha$$
$$\frac{x}{v_0 cos\alpha} = t$$

Z-Direction

$$y = y_0 + v_{0y}t + \frac{at^2}{2}$$
$$y = v_0 t sin\alpha - \frac{gt^2}{2}$$

Note that $y = xtan\beta$. Now using the the equation for time found with the x direction equations of motion with the y position function we get

$$
\begin{aligned}
y = xtan\beta =& v_0 sin\alpha t - \frac{gt^2}{2} \\
xtan\beta =& v_0 \frac{x}{v_0 cos\alpha} sin\alpha - \frac{g\left(\frac{x}{v_0 cos\alpha}\right)^2}{2} \\
xtan\beta - \frac{x}{cos\alpha} sin\alpha =& - \frac{g\left(\frac{x}{v_0 cos\alpha}\right)^2}{2} \\
xtan\beta - xtan\alpha =& \frac{gx^2}{2v_0^2 cos^2\alpha} \\
x\left[tan\alpha - tan\beta\right] =& \frac{gx^2}{2v_0^2 cos^2\alpha}
\end{aligned}
$$

Where here we use the trigonometric identity

$$tanx - tany = secxsecysin(x-y)$$

And insert this back into the previous step.

$$
\begin{aligned}
\frac{gx^2}{2v_0^2 cos^2\alpha} =& \left[tan\alpha - tan\beta\right] \\
\frac{gx}{2v_0^2 cos^2\alpha} =& \left[\frac{sin(\alpha-\beta)}{cos\alpha cos\beta}\right] \\
x =& \frac{2v_0^2 cos^2\alpha}{g}\left[\frac{sin(\alpha-\beta)}{cos\alpha cos\beta}\right] \\
x =& \frac{2v_0^2}{g}\left[\frac{sin(\alpha-\beta)cos\alpha}{cos\beta}\right]
\end{aligned}
\tag{8}
$$

3.0.6 Part B

To determine the maximum range range we must take the derivative of the range function with respect to the parameter α, then set it equal to zero.

$$
\begin{aligned}
\frac{dx}{d\alpha} = 0 =& \frac{2v_0^2}{gcos\beta}\left[cos(\alpha-\beta)cos\alpha - sin(\alpha-\beta)sin\alpha\right] \\
0 =& cos(\alpha-\beta)cos\alpha - sin(\alpha-\beta)sin\alpha \\
cos(\alpha-\beta)cos\alpha =& sin(\alpha-\beta)sin\alpha \\
1 =& tan(\alpha-\beta)tan(\alpha) \\
1 =& cos(2\alpha-\beta) \\
cos^{-1}(1) =& 2\alpha-\beta \\
\pi =& 2\alpha-\beta \\
\alpha =& \frac{\pi}{2} + \frac{\beta}{2}
\end{aligned}
\tag{9}
$$