Physics 5B Practice Problems 3

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1 Multiple Choice Section

1. A fresh-pure-water ice cube is floating in a glass filled to the rim with salty sea water. When the ice melts, the water

   A. goes down

   B. over flows

   C. stays the same

2. A 200-ton ship enters a canal. The fit between the sides of the lock and the ship is tight, so that the weight of the water left in the lock after it closesis much less than 200 tons. Can the ship still float in water?

   A. Yes, as long there is enough water to reach up to the ships waterline (the level of the water surface relative to the ship when it is floating in open sea).

   B. No, the ship touches the bottom because it weighs more than the water in the lock.

3. A speaker emits sound uniformly in all directions. Point B is half as far from the speaker as is Point A.The sound intensity level at Point B is

   A. 3 dB more than at Point A.

   B. 6 dB more than at Point A.

   C. 10 dB more than at Point A.

   D. 20 dB more than at Point A.

4. A thin film is applied to the surface of a window in order to maximize the reflection of infrared light from the surface (while still allowing visible light to pass through). The film has a thickness equal to 1/4 the wavelength of the infrared light (in the film). The index of refraction of the film should be

   A. less than that of the glass.

   B. equal to that of the glass.

   C. greater than that of the glass.

   D. the reciprocal of that of the glass.

5. A wave pulse is sent down a rope made by tying a thin, light rope to a thick, heavy rope. The pulse starts in the heavy section. When it reaches the boundary between thin and heavy section part of the pulse will reflect. With respect to the incident pulse, the reflected pulse will

   A. inverted.

   B. not inverted.

   C. It will not reflect

   D. Not enough information given

6. A mechanic using a manual hydraulic jack to lift a car of weight 27,000 N pushes on a reservoir of oil using a piston of radius 1 cm. The car is lifted by the same oil pushing on a piston of radius 30 cm. With how much force F must the mechanic push in order to lift the car quasi-statically?

   A. 30 N

   B. 300 N

   C. 900 N

   D. 13,500 N

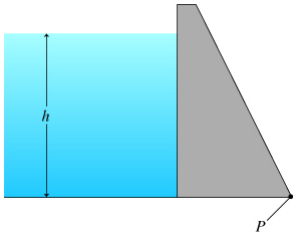
7. In the example of the previous problem, what distance must the mechanic push the small piston in order to lift the car 3cm? A. 1cm B. 30cm C. 900cm **D. 2700cm**

2 Free Response Section

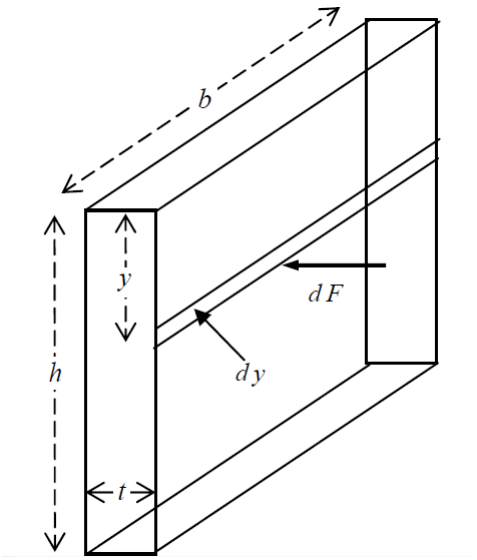
8. As the reservoir behind a dam is filled with water, the pressure that the water exerts on the dam increases. Eventually, the force on the dam becomes substantial, and it could cause the dam to collapse. There are two significant issues to be considered: First, the base of the dam should be able to withstand the pressure $\rho g h$, where ρ is the density of the water behind the dam, h is its depth, and g is the magnitude of the acceleration due to gravity. This means that the material of which the dam is made needs to be strong enough so that it doesn't crack (compressive strength). The second issue has to do with the strength of the foundation of the dam. The water pressure exerts a clockwise torque on the dam, as shown in the figure. The foundation of the dam should be strong enough so that the dam does not topple. The material has to be strong enough that the dam does not snap (shear strength). To study this phenomenon, consider the simple model of a dam depicted in the diagram. A reservoir of water (density) behind the dam is filled to a height . Assume that the width of the dam (the dimension pointing into the screen) is L.

   A. Calculate the total force on the dam due to the water in the reservoir.

   B. The force of the water produces a torque on the dam. In a simple model, if the torque due to the water were enough to cause the dam to break free from its foundation, the dam would pivot about its base (point P). What is the magnitude τ of the torque about the point P due to the water in the reservoir?



Solution:



Part A Because the pressure varies with depth, the force on the wall will also vary with depth. So to find the total force on the wall, we will have to integrate. Measure vertical distance y downward from

the top level of the water behind the dam. Then at a depth y, choose an infinitesimal area of width b and height dy. The pressure due to the water at that depth is $P = \rho g y$.

$$
\begin{aligned}
dF =& PdA = (\rho g y)(b dy) \longrightarrow \\
F =& \int dF = \int_0^h (\rho g y)(b dy) \\
F =& \rho g b \int_0^h y dy = \rho g b \left[\frac{y^2}{2} \right]_0^h \\
F =& \frac{1}{2} \rho g b h^2
\end{aligned}
$$

Part B The lever arm for the force dF about the bottom of the dam is $h - y$, so the torque caused by that force is $d\tau = (h - y)dF$. Integrate to find the total torque.

$$
\begin{aligned}
\tau =& \int d\tau \\
\tau =& \int_0^h (h - y)(\rho g y)(b dy) \\
\tau =& \rho g b \int_0^h (hy - y^2) dy \\
\tau =& \rho g b \left[\frac{1}{2} h y^2 - \frac{1}{3} y^3 \right]_0^h \\
\tau =& \frac{1}{6} \rho g b h^3
\end{aligned}
$$

9. Air streams horizontally across an aeroplane wing of area $4m^2$, weighing 300 kg. The air speed is 70 m/s and 55 m/s over the top surface and the bottom surface, respectively. Find (a) the lift on the wing; (b) the net force on it. Hint: The lift is defined as the pressure difference times the area of the wing.

Solution: Part A

$$\begin{aligned}\Delta P =& \frac{1}{2}\rho(v_1^2 - v_2^2) \\ =& \frac{1}{2} \times 1.293 \times (70^2 - 55^2) = 1212Pa\end{aligned}$$

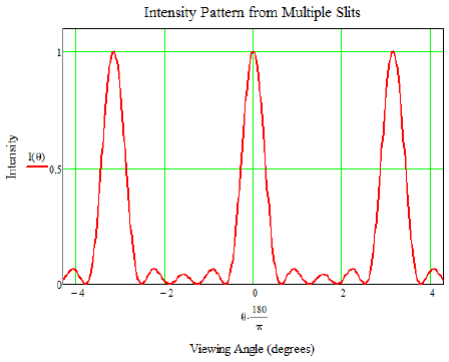
Now, lift is defined by the pressure difference times the area of the wing. Thus, the lift is

$$Lift = 1212Pa \times 4m^2 = 4848N$$

Part B The net force is just the lift minus the weight of the plane. Hence,

$$\sum F = 4848N - (300 \times 9.8m/s^2)$$

10. The graph shows the intensity pattern of interference from laser light with $\lambda = 600nm$ shining through several very narrow slits and onto a screen.



a) How many slits N are there? b) Draw a phasor diagram showing how the N equal amplitudes add together as vectors to produce the first minimum just to the right of the central maximum. What is the phase difference between consecutive phasors in your diagram? c) Draw a phasor diagram showing how the N equal amplitudes add together as vectors to produce the smallest maximum (half way between principal maxima). What is the phase difference between consecutive phasors in your diagram? d) Given that the smallest maximum occurs at an angle of 1.58 degrees, calculate the distance between successive slits. e) Explain why the smallest maximum has exactly 1/25 the intensity of the central maximum

Part A: How many slits N are there?

Solution:

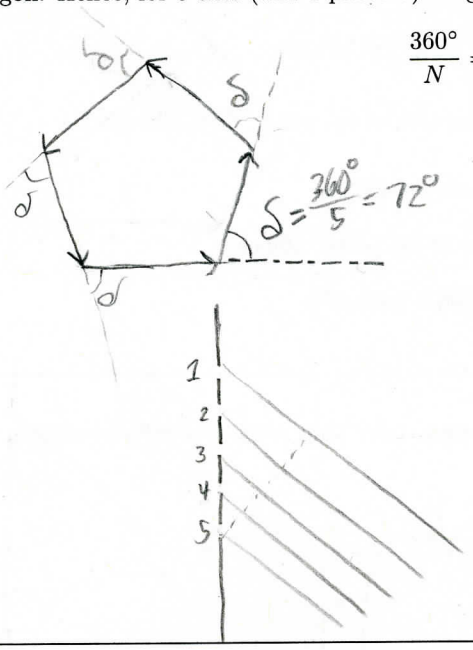
There are as many slits as there are peaks in between (and including) the two absolute maximum. Hence,

$$N = 5$$

Part B: Draw a phasor diagram showing how the N equal amplitudes add together as vectors to produce the first minimum just to the right of the central maximum. What is the phase difference between consecutive phasors in your diagram?

Solution: The only way you can add up all 5 phasors to equal the zero phasor is if you create a closed polygon. Hence, for 5 slits (and 5 phasors) we get a pentagon with phase of

$$\frac{360°}{N} = \frac{360°}{5} = 72°$$



δ is the phase of the electric field amplitude of the light being diffracted from one slit relative to the previous slit (ie. in between slit 1 and 2, or 3 and 4).

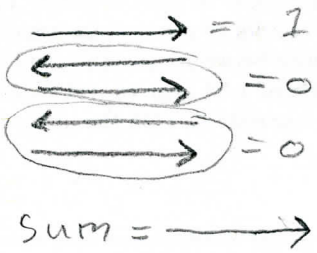
The electric field is described by $E(x,t) = E_0 \cos(kx - \omega t + \delta)$, but we are only concerned with one x position.

$\Rightarrow E(x=x_0, t) = E_0 \cos(\omega t + \delta)$

Part C: Draw a phasor diagram showing how the N equal amplitudes add together as vectors to produce the smallest maximum (half way between principal maxima). What is the phase difference between consecutive phasors in your diagram?

Solution: The smallest maximum (for odd number of slits) occurs when all except one phasor remains. Hence, this is when each phasor is 180° out of phase or

$$\delta = \pi$$



Part D: Given that the smallest maximum occurs at an angle of 1.58 degrees, calculate the distance between successive slits.

Solution: Since we know the phase difference δ for the smallest maximum, and we know the angle where that occurs we just use algebraically manipulate our phase equation.

$$\delta = \frac{2\pi d}{\lambda} sin\theta \longrightarrow d = \frac{\delta\lambda}{2\pi sin\theta}$$

But from part C we calculated $\delta = \pi$ therefore,

$$d = \frac{\pi\lambda}{2\pi sin\theta} = \frac{600 \times 10^{-9} m}{2sin(1.58°)} \approx 10 \times 10^{-6} m$$

Part E: Explain why the smallest maximum has exactly 1/25 the intensity of the central maximum.

Solution: The principal maximum phasor amplitude is

$$\longrightarrow\longrightarrow\longrightarrow\longrightarrow\longrightarrow$$

Which is 5 times larger than smallest maximum phasor amplitude of $\longrightarrow$. Since the phasor represents the electric field of the electromagnetic rays (the coherent light) then the intensity is proportional to the square of the electric field amplitude. Therefore, $I_{min} = \frac{I_0}{25}$