Physics 5A Review 1

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1 Error, Sig Figs, and Dimensional Analysis

If we have measurements 8.31, 8.35, and 8.41 we get an average of 8.36 with an error of ± 0.05. The basic rule of thumb is first take into account the error in the measurements, then look at the significant figures.

For significant figures all we do is first look at each *measured values* that are given, but not the given constants. Then we find the value with the least significant figure and use that as how many sig figs you will quote. If we have the measured values: 320, 20.1 , 4.1213, and 0.0013345.The least sig fig out of these numbers is 320 with two sig figs. Our final value would be 1500 or 32 or 0.00000095 or any number with two sig figs.

Dimensional analysis is a way of changing units. Therefore we must have some conversion ratios to actually use dimensional analysis. The way we do this is by taking our units and multiplying it by the conversion ratio. As an example I will convert from kilograms to Eric's.

$$1kg \times \frac{10Freds}{1kg} \times \frac{20Jams}{10Freds} \times \frac{1Eric}{2Jams} = 10Erics$$

2 Vector Review

A vector is a mathematical tool used to represent physical quantities. Vectors are extremely useful for physics (and necessary!) to describe natural phenomena. For example, if I say there is two forces acting on a particle and then ask you to calculate the resulting acceleration, you wont be able to. Why? Because the acceleration is dependent on the direction of the net force and on its direction. To overcome this problem we use vectors. A vector just gives us a direction and magnitude of the quantity in question.

I can represent a vector in multiple ways. First I could say the first force is directly upwards, and the second force is directly downwards, and that they have equal magnitudes. Now you could represent these vectors as

$$\vec{F}_1 = \left(0 \cdot \hat{\imath} + F \cdot \hat{\jmath}\right) \qquad \vec{F}_2 = \left(0 \cdot \hat{\imath} - F \cdot \hat{\jmath}\right)$$

To calculate the acceleration we need to find the sum of the forces on the body and then divide by mass (see section 5, or chapter 4 in Giancoli, for further details). So now we need to know how to add vectors.

2.1 Adding/Subtracting Vectors

To add two vectors together we just add each corresponding component of the vector together. Using the vectors from the example we will obtain

$$\vec{F}_1 + \vec{F}_2 = \left(0 \cdot \hat{\imath} + F \cdot \hat{\jmath}\right) + \left(0 \cdot \hat{\imath} - F \cdot \hat{\jmath}\right) = \left(0 \cdot \hat{\imath} + 0 \cdot \hat{\imath} + F \cdot \hat{\jmath} + -F \cdot \hat{\jmath}\right) = \left(0 \cdot \hat{\imath} + 0\hat{\jmath}\right)$$

If we wanted to subtract to vectors all we would do is multiply each component of the vector being subtracted then add them together. The same process is used for adding vectors and subtracting vectors.

We can characterize this process to as many vectors as we want using summation notation as

$$\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \cdots + \vec{v}_n = \sum_{m=0}^{n} \vec{v}_m = \left(\sum_{m=0}^{n} v_{mx}\hat{\imath} + \sum_{m=0}^{n} v_{my}\hat{\jmath} + \sum_{m=0}^{n} v_{mz}\hat{k}\right) \qquad (1)$$

2.2 Multiplying Vectors: Dot and Cross Products

There are two operations we use when multiplying vectors together. I will omit all theory and in-depth geometric discussion about these two types of vector multiplication. However, I will discuss their relations to physics.

2.2.1 Dot Product or Scalar Product

Dot product produces a scalar value. It represents the amount of one vector that is in the direction of another vector. For example, if we wanted to calculate the work done on a particle by moving it through some force field we would use the dot product. Why you ask? Because work is done only when the force is the direction of motion. Its like a air hockey table, the puck moves around without any push from you because the air blowing up on it effectively gets rid of friction, and the force of gravity only acts downward. Hence, the work needed to move the puck is zero since its direction of motion is perpendicular to the direction of the force.

The dot product is computed by multiplying the corresponding components of the two vectors together.

$$\vec{v}_1 \cdot \vec{v}_2 = \left(v_{1x} v_{2x} \hat{i} \cdot \hat{i} + v_{1y} v_{2y} \hat{j} \cdot \hat{j} + v_{1z} v_{2z} \hat{k} \cdot \hat{k} \right) = v_{1x} v_{2x} + v_{1y} v_{2y} + v_{1z} v_{2z} \tag{2}$$

2.2.2 Cross Product or Vector Product

$$\vec{r} \wedge \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = \hat{i} \left(F_z r_y - F_y r_z \right) - \hat{j} \left(F_z r_x - F_x r_z \right) + \hat{k} \left(F_y r_x - F_x r_y \right)$$

The magnitude of a cross product is defined as

$$|r \wedge F| = |r||F| sin\theta$$

3 One Dimensional Kinematics

3.1 Displacement vs. Distance

Distance is a scalar measure of the interval between two locations measured along the actual path connecting them.
Displacement is a vector measure of the interval between two locations measured along the shortest path connecting them.

If you walk to the left 20 meters then return to the position you started at your displacement is 0 meters, and the distance traveled is 40 meters.

3.2 Velocity vs. Speed

Velocity is the rate of change of displacement per unit time. This implies that velocity is a vector. There can be a change in velocity without a change in speed, simply by changing direction.

Speed is the rate of change of the distance. This implies that speed is a scalar.

3.3 Constant Acceleration Kinematics

We live in gravitational field produced by the mass of earth. We assume that it is constant $9.8\frac{m}{s^2}$ near the surface of the earth. We define this acceleration as the constant g. We say that the acceleration in the gravitational field is the rate of change of the velocity, $a = \frac{dv}{dt} = 9.8\frac{m}{s^2}$. Now we just integrate to obtain the velocity equation of motion

$$\begin{aligned} \frac{dv}{dt'} &= a \\ \int_{v_0}^{v(t)} dv &= \int_0^t a dt' \\ v(t) - v_0 &= at' \mid_0^t \\ v(t) &= v_0 + at \end{aligned} \tag{3}$$

Following the same process, with $v(t) = \frac{dx}{dt}$ we obtain the position as a function of time as

$$\begin{aligned} \frac{dx}{dt'} &= v_0 + at \\ \int_{x_0}^{x(t)} dx &= \int_0^t v_0 + at' dt' \\ x(t) - x_0 &= v_0 t' + \tfrac{1}{2} a t'^2 \mid_0^t \\ x(t) &= x_0 + v_0 t + \tfrac{1}{2} a t^2 \end{aligned} \tag{4}$$

3.3.1 Key Kinematic Equations

1. $a(t) = a$
2. $v(t) = v_0 + at$
3. $x(t) = x_0 + v_0 t + \frac{at^2}{2}$
4. $v^2(t) = v_0^2 + 2a\left(x(t) - x_0\right)$

3.4 Example: Train Distance

A train at station A starts from rest at $t = 0$ and accelerates at $5\frac{m}{s^2}$ for 10 seconds. It remains at this velocity for one hour then decelerates at -$4\frac{m}{s^2}$. (a) What is the maximum velocity reached? (b) How long does it take to come to a complete stop, and how far does it travel in that time? (c) What is the total displacement of the train? (d) Graph the position, velocity, acceleration as a function of time.

3.4.1 Part A

Since we are given the acceleration of the train we just integrate acceleration function with respect to time (acceleration as a function of time is constant). The final velocity after this acceleration would give us the maximum velocity since there is no more positive acceleration after $t = 10s$. We would obtain Equation 1 with $v_0 = 0$ hence

$$\begin{aligned}
v(t = 10s) &= v_0 + at \\
v(t = 10s) &= 0\tfrac{m}{s} + 5\tfrac{m}{s^2} \times 10s \\
v(t = 10s) &= 50\tfrac{m}{s}
\end{aligned}$$

The maximum velocity (and speed) is $50\frac{m}{s}$. It will remain at $50\frac{m}{s}$ for the next hour.

3.4.2 Part B

For the train to come to a complete stop means $v(t) = 0$. We know the initial velocity to be $50\frac{m}{s}$ as found in part A. We integrate the acceleration function (a constant) which is results in Equation 1 with $v(t) = 0$ and $v_0 = 50\frac{m}{s}$.

$$\begin{aligned}
v(t) &= v_0 + at \\
0 &= 50\tfrac{m}{s} - 4\tfrac{m}{s^2} \times t \\
4\tfrac{m}{s^2} \times t &= 50\tfrac{m}{s} \\
t &= \frac{50\frac{m}{s}}{4\frac{m}{s^2}} \\
t &= 12.5s
\end{aligned}$$

Now that we know how long it took to come to a complete stop we can determine how far it traveled during that time. Integrating our velocity function we obtain Equation 2 with $x_0 = 0$, since we are only interested in the distance it traveled from when it began decelerating.

$$\begin{aligned}
x(t = 12.5s) &= 0 + 50m/s12.5s + \tfrac{1}{2}(-4\tfrac{m}{s^2})(12.5s)^2 \\
x(t = 12.5s) &= 625m + 312.5m \\
x(t = 12.5s) &= 937.5m
\end{aligned}$$

The train travels $937.5m$ during the time time it takes to stop.

3.4.3 Part C

We have to break up the trip into three segments since acceleration is not constant throughout the trip. The segments are as follows: first where the train accelerates, second where the train moves with constant velocity, third where the train decelerates to rest.

The first distance is given by Equation 2 as

$$\begin{aligned}
x(t = 10s) &= 0m + 0\tfrac{m}{s}10s + \tfrac{1}{2}(5\tfrac{m}{s^2})(10s)^2 \\
x(t = 10s) &= 250m = 0.25km
\end{aligned}$$

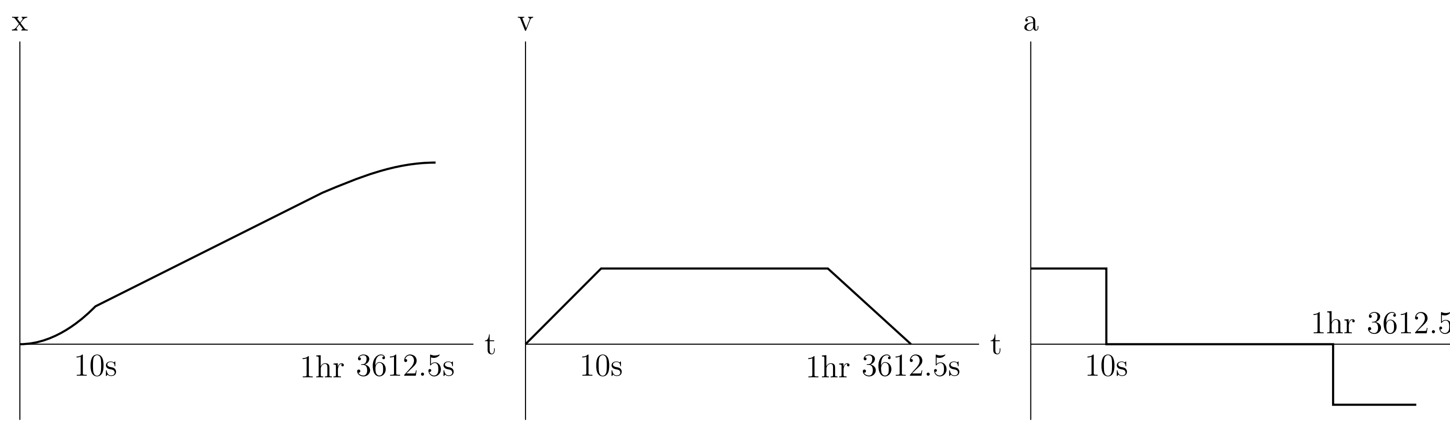
The second distance is given by integrating Equation 1, or Equation 2 with $a = 0$, since there is no acceleration.

$$x(t = 3600s) = 0m + 50\frac{m}{s}3600s + \frac{1}{2}(0\frac{m}{s^2})(3600s)^2$$
$$x(t = 3600s) = 180,000m = 180km$$

The third distance will be determined in the same way as the first distance. We have already determined how far the train will travel during the deceleration in Part B, which was $937.5m$. Now all we do is add all the distances together to determine the total distance.

$$x_{total} = 250m + 180,000m + 937.5m = 181,187.5m = 181.1875km$$

3.4.4 Part D



4 Two Dimensional Kinematics

The same rules apply for kinematics in two dimensions (and three dimensions). We generally break kinematic problems apart into components, the x-direction and y-direction (and the z-direction). For projectile motion there is no acceleration in the x-direction, unless you consider air resistance [1]. From the figure below we see that only the vertical component of velocity changes magnitude, hence there is only vertical acceleration (or deceleration). This makes sense because gravity is the only force acting on the projectile (excluding air resistance) and it always points downward.

[1]For this class and almost all introductory physics courses we consider air resistance to be negligible.

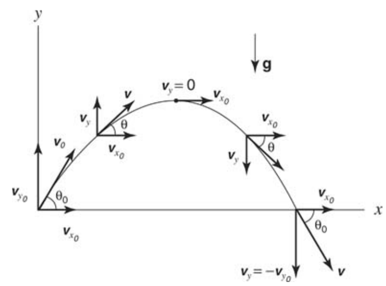


Figure 1: The components of velocity of a projectile. The motion is parabolic. Air resistance is negligible.

To determine the velocity or position as a function of time we must integrate the acceleration function. For most kinematics problems the acceleration is just gravity which is a constant (near the earths surface). The velocity function and position function, respectively, are

- $v(t) = \int^t a(t)dt + v_0$
- $x(t) = \int^t \left[\int^t a(t)dt + v_0\right] dt + x_0$

Where the constants of integration, v_0 and x_0, are the initial velocity and initial position, respectively.

4.1 X-Direction Equations

4.1.1 X-Acceleration

Unless there is air resistance or some kind of other external force acting on the projectile there is no acceleration in the x-direction.

4.1.2 X-Velocity

The velocity in the x-direction is, by definition of no acceleration, constant during the entire trajectory. The equation for velocity in the x-direction as a function of time

$$v_x(t) = v_0 cos(\theta)$$

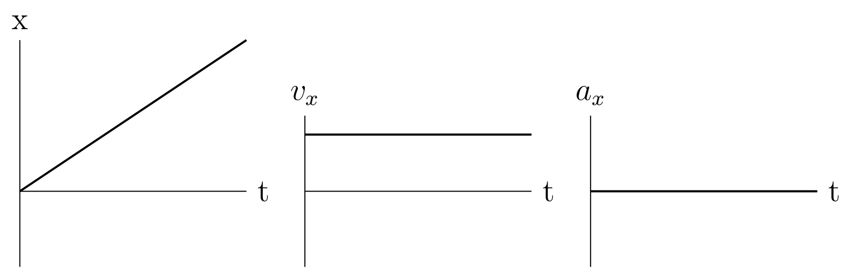
Where θ is the angle from the horizontal that the projectile is launched from, and v_0 is the initial velocity.

4.1.3 X-Position

Since the velocity in the x-direction is constant the position of the projectile increases proportionally to time. Depending on how much initial velocity the projectile is given will determine how rapidly it gains distance. Therefore the x-position as a function of time is given by

$$x(t) = v_x t$$

4.1.4 X-Graphs



4.2 Y-Direction Equations

4.2.1 Y-Acceleration

For all projectile motion problems there will be gravity acting on the object during its entire trajectory. Gravity always points downward and is constant throughout the duration of motion. It is expressed as

$$a = g = 9.8\frac{m}{s^2}$$

This value will only change when on a different planets surface (i.e. the moon, mars, venus, etc) or when very far from the earths surface. However, there is a different approach to solving problems very far from the earths surface. These are often rockets and satellites.

4.2.2 Y-Velocity

The velocity in the y-direction starts from a maximum (if we assume motion is launched above the horizontal) then decreases until it reaches the apex of the trajectory. At the apex it has zero velocity in the vertical direction. This is where the velocity changes from positive to negative [2]. Remember that acceleration is constant, it is the same when it has maximum positive velocity as when it has no velocity or negative velocity. To describe the velocity as a function of time we use Equation 1

$$v(t) = v_0 + at$$

We see that this is a straight line with v_0 as the "y-intercept" and the acceleration, usually g, is the slope of the line. This equation works for any situation of projectile motion.

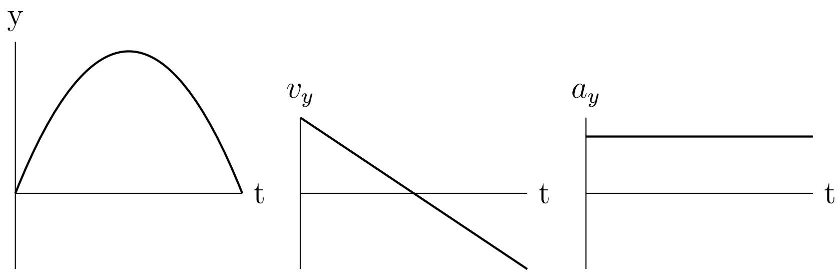
4.2.3 Y-Position

The vertical position of the projectile increases to a maximum then decreases. It is parabolic motion, just like the equation describing it

$$y(t) = y_0 + v_0 sin(\theta)t + \frac{1}{2}a_y t^2$$

[2] Or it changes from negative to positive depending on the coordinate axes you chose.

4.2.4 Y-Graphs



4.3 Cool Facts

- A ball thrown upward with initial velocity v_0 reaches a max height $x = \frac{v_0^2}{2g}$ and has a final velocity when it reaches the ground of $-v_0$.
- The same ball will take a time $t = \frac{v_0}{g}$ to reach max velocity. This found by knowing that at t the velocity is zero and using our velocity equation.
- A bullet shot horizontally at $1000\frac{m}{s}$ takes the same amount of time to hit the ground as the shell casing. If we neglect air resistance.
- If you dropped a ball from a cliff, of height H and at the same time your friend through a ball upwards with initial velocity v_0, the balls would collide (if they did at all) at time $t = \frac{H}{v_0}$. This makes sense if you put yourself in accelerating reference frame...

4.3.1 Monkey and the Hunter

Remember the demonstration of the monkey dropping and the hunter (you blowing a ball through a tube), the monkey and the ball fell downward at the same rate even though the ball had a large x-velocity. Here's a picture of the setup.

4.3.2 Relative Velocity

If a rain falls down at $1\frac{m}{s}$ and you are driving in the positive x direction at $5\frac{m}{s}$. The rain on your window will make an angle with the negative x-axis $\theta = tan^{-1}\left(\frac{v_{rain}}{V_{car}}\right)$. Using this idea we can find the velocity of the rain if we measure the angle θ and know how fast the car is going. From this we find that the velocity of rain is given by

$$v_{rain} = v_{car} tan\theta$$

5 Dynamics

We explained how objects move using kinematics, now we will describe why objects move using dynamics. There are three laws of dynamics which were postulated by Isaac Newton. The laws are as follows

1. An object in motion tends to stay in motion unless otherwise acted upon by an external force. Similarly an object at rest tends to remain at rest unless acted upon by an external force.

2. An external force acting upon an object is proportional to the acceleration and is dependent on the mass. The force is also equivalent to the instantaneous rate of change of the momentum. Force is also equivalent to the negative of the instantaneous rate of change of potential energy.

3. Every force (action) has an equal and opposite force (reaction).

Basically, all of classical mechanics is wrapped up in these three laws (mainly the second law). There are few important techniques and concepts that will be applied in every mechanics problem.

5.1 Coordinate axes

We do not necessarily have to choose the x-axis going right and y-axis up. In fact it is often easier to *not* choose the standard coordinate system. For inclined planes we tend to put the y-axis in the direction of the normal force, as seen in the figure below. This would put the x-axis entirely in the direction of motion, and hence the direction of acceleration. By putting one of the coordinate axes in the direction of acceleration. This makes the equations simpler.

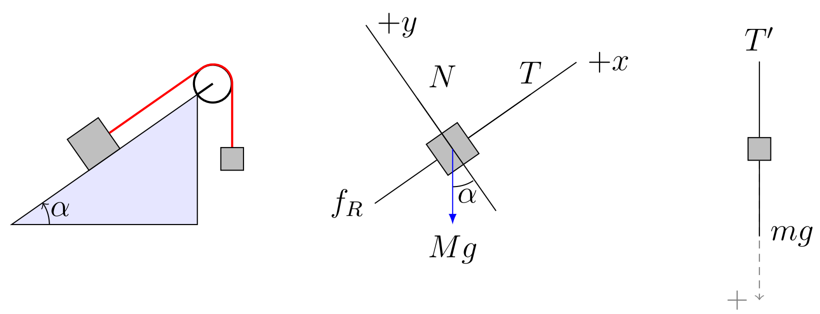


Figure 2: We choose two different coordinate axes for each box in this system. The box on the inclined plane has a tilted coordinate axes, while the hanging mass has positive downward.

It is important to note that for two body systems we can can choose two different coordinate systems, we just have to stay consistent with our equations.

Please note that it is very convenient to put one axis in the direction of acceleration. This will give you two equations of Newtons second law (one for x-direction and one for y-direction) in which one of them is zero.

5.2 Free-Body-Diagrams

The free body diagram is easily the most important idea stemming from Newtons Laws. It allows us to clearly label all the forces on the object in question. This in turn gives us an easy way to write our equations describing motion. There is no difference between the equations and the free body diagrams (FBD). The FBD shows us the equations to use. They are merely two different ways of expressing physical phenomena. A free body diagram is shown in Figure 2 above (middle and left objects).

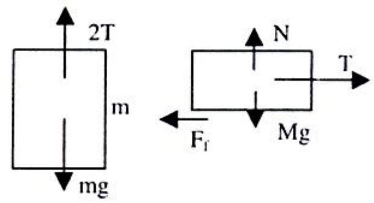


Figure 3: A free body diagram. There are forces in the x-direction and the y-direction. We look at each direction separately when using Newtons second law.

To use a FBD we first determine what forces are acting on the object. We ask ourselves: Is there friction? Force of gravity? Tension? etc. We then label these forces on the FBD in the direction that the force is being applied.

I am sitting in a chair right now. The chair is pushing up with a normal force that is counter acting the force due to my mass in earths gravitational field. The FBD of situation would be a box, circle, dot -something simple to represent myself. I would then draw an arrow upwards representing the force applied by the chair called the normal force. Then I would draw an arrow downward representing the force due to my weight. These arrows should be equal and opposite if I were not accelerating. If I was in an adjustable height chair and slowly decreasing the height then the arrows would not be equal. What arrow would longer? The force due to my weight would keep the same magnitude, hence the arrow would stay the same length. However, the the normal force would decrease. It would decrease because I am accelerating downward. This means there is a net force on my body downwards. This is the process for every problem encountered for this section. There are two main forces we will look at

5.2.1 Weight and Normal Force

Weight and normal force are an example of Newtons third law. The weight of an object is an action and the normal force is the reaction. Every object that is at rest relative to the earths surface has a normal force. If it did not have a normal force it would not be at rest. The normal force holds your textbook on the shelf, phone on the table car on the ground,etc. Please note that the normal force is not always equal in magnitude to the weight of the object. It also controls how much your car grips the ground through the force of friction.

5.2.2 Friction

Friction is always a retarding force. It acts in opposition to motion. Friction is caused by the microscopic irregularities on two surfaces that come into contact. These forces are ultimately caused by electromagnetic interactions between electrons of the atoms making up the two different objects. There are two types of friction static and kinetic. Static friction is the friction keeping an object from sliding around. It is stronger than kinetic energy. That is why it is harder to push a heavy box starting at rest than after it has begun moving. The amount of static friction is determined experimentally for each material. But is determined by a coefficient and the normal force. The amount of friction, static or kinetic, is directly proportional to these two quantities.

$$F_s \leq \mu_s N \tag{5}$$

Where N is normal force and μ_s is the coefficient of static friction. there is a les than or equal to sign because friction is only as big as it needs to be. Otherwise, objects would fly off tables left and right due to friction!

Kinetic friction is always less than static friction for the same material. The equation for kinetic friction is similar to static friction except there is no less than part. The coefficient of kinetic friction is a constant.

$$F_k = \mu_k N \tag{6}$$

Where N is normal force and μ_k is the coefficient of kinetic friction. The graph as a function of applied force to an object looks like the following function of frictional force vs applied force.

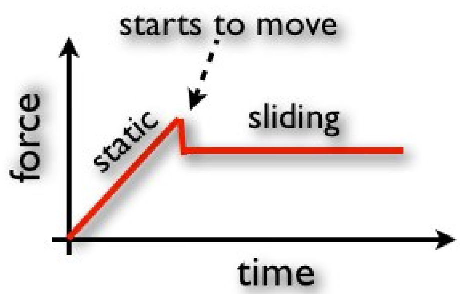


Figure 4: During the static region the object your applying a force has not moved. Once it reaches the boundary between static and kinetic (sliding) it overcomes the electromagnetic forces between electrons

5.3 Uniform Circular Motion

Uniform circular motion is weird because the particle/object has a constant tangential velocity, and constant angular velocity. Since velocity is a vector the object is actually accelerating. Not because its increasing its speed but because its changing its direction. We call this type of acceleration centripetal acceleration. I will omit the derivation because the text has a very nice derivation. The centripetal acceleration is given by

$$a_c = \frac{v^2}{r} = \omega^2 r = v\omega \tag{7}$$

Where we have used the relations and can use the following relations to obtain different quantities

$$v = \omega r \tag{8}$$

$$\omega = \frac{2\pi}{T} = 2\pi f \tag{9}$$

$$v = \frac{2\pi r}{T} = 2\pi r f \tag{10}$$

5.3.1 Uniform Circular Motion and Newton's Second Law

We can use Newton's second law to determine the motion of an object by using our centripetal acceleration in the second law

$$\sum F = ma_c = m\frac{v^2}{r} \tag{11}$$

Then we can carry out our calculations as in other dynamics problem. Please note that there must be some NET force (tension, gravity, friction, etc.) keeping the object moving in the circular. And this equation only works for uniform circular motion, where uniform just means constant speed.

6 Gravitation

Everything that has mass has a gravitational field. You have a gravitational field because you have mass. The earth, moon, all planets, stars galaxies all have mass. Gravity is the weakest of the four fundamental forces, yet it is the dominant force in the universe for shaping the large scale structure of galaxies, stars, etc. The gravitational force between two masses m_1 and m_2 is given by the relationship:

$$F_g = G\frac{m_1 m_2}{r^2}\hat{r} \tag{12}$$

Where $G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$ and is called universal gravitational constant. This is often called the "universal law of gravitation" and G the universal gravitation constant. It is an example of an inverse square law force. The force is always attractive and acts along the line joining the centers of mass of the two masses. The forces on the two masses are equal in size but opposite in direction, obeying Newton's third law. Viewed as an exchange force, the mass-less exchange particle is called the "graviton".

Any point source which spreads its influence equally in all directions without a limit to its range will obey the inverse square law. This comes from strictly geometrical considerations. As one of the fields which obey the general inverse square law, the gravity field can be put in the form shown below, showing that the acceleration of gravity, g, is an expression of the intensity of the gravity field.

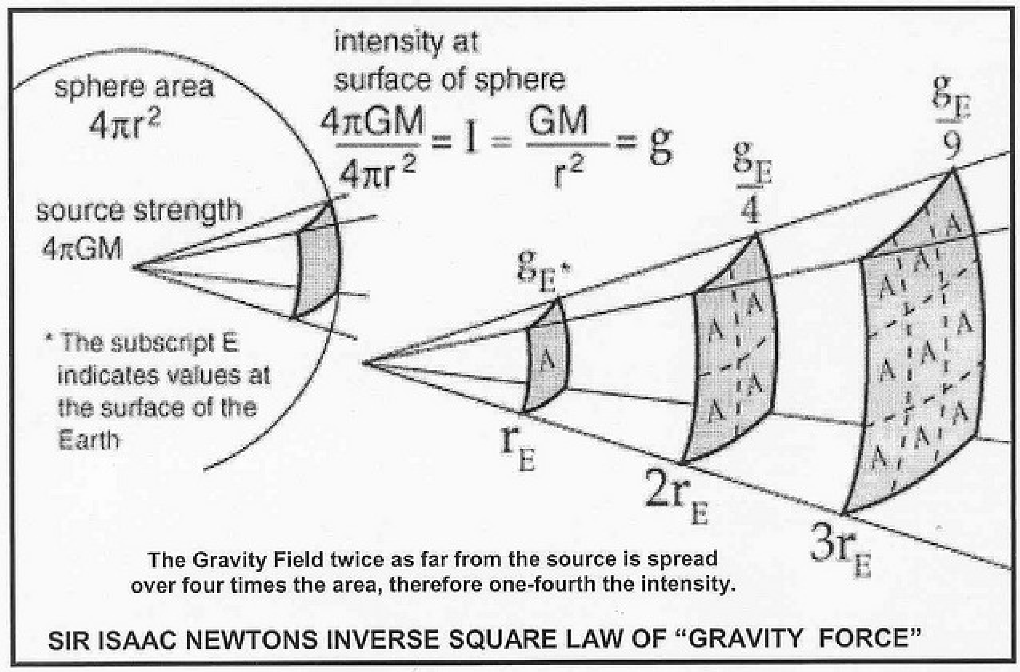


Figure 5: This is a graphical representation of how gravity permeates space time.