Physics 139A Homework 2

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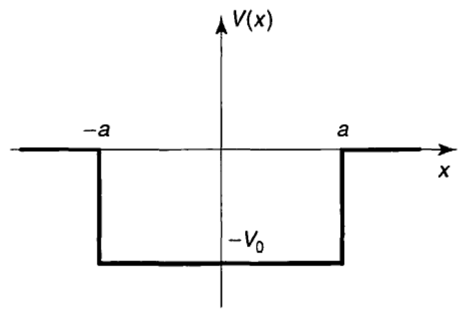
1 Problem 2.29

Analyze the odd bound state wavefunctions for the finite square well. Derive the transcendental equation for the allowed energies, and solve it graphically. Examine the two limiting cases. Is there always an odd bound state?



1.1 Solution to Problem 2.29

There are three regions for the finite square well. The two regions that are outside the well, $|x|> a$, and the region that is inside the well, $|x|\leq a$. Let us write out Schrodinger's equation for both inside and outside the well. Note that since the particle is bound to within the well its overall energy is $E < 0$. I will right E as the magnitude and carry around the negative value.

Outside the well:

$$\begin{aligned}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) &= -E\psi(x)\\ \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) - E\psi(x) &= 0\\ \frac{\partial^2}{\partial x^2}\psi(x) - \frac{2mE}{\hbar^2}\psi(x) &= 0\\ \frac{\partial^2}{\partial x^2}\psi(x) - \kappa\psi(x) &= 0\end{aligned}$$

With $\kappa = \sqrt{\frac{2mE}{\hbar^2}}$
and solutions : $\psi(x) = Ae^{\kappa x} + Be^{-\kappa x}$

Inside the well:

$$\begin{aligned}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) - V_0\psi(x) &= -E\psi(x)\\ \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + (V_0 - E)\psi(x) &= 0\\ \frac{\partial^2}{\partial x^2}\psi(x) + \frac{2m(V_0 - E)}{\hbar^2}\psi(x) &= 0\\ \frac{\partial^2}{\partial x^2}\psi(x) + \gamma^2\psi(x) &= 0\end{aligned}$$

With $\gamma = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$
and solutions : $\psi(x) = Csin(\gamma x) + Dcos(\gamma x)$

Now physically the wavefunction must go to zero when outside the well, for obvious reasons. This must mean that for $x < -a$ then $A = 0$, and for $x > a$ then $B = 0$. Since we are looking for the odd wavefunctions we will take $D = 0$. The overall wavefunction is then just

$$\psi(x) = \begin{cases} Ae^{\kappa x}, & \text{if } x < -a \text{ with } \kappa = \sqrt{\frac{2mE}{\hbar^2}} \\ Csin(\gamma x), & \text{if } |x| \leq a \text{ with } \gamma = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} \\ Ae^{-\kappa x}, & \text{if } x > +a \text{ with } \kappa = \sqrt{\frac{2mE}{\hbar^2}} \end{cases}$$

Now that we have the general wavefunction we must now impose boundary conditions. At the walls of the well the wavefunction should be smooth and continuous. However, since the wavefunction is odd we can just look at the $x = +a$ boundary (or the $x = -a$ boundary) and we know that the other boundary is also smooth and continuous. The boundary conditions for $x = +a$ are

Continuous Condition:

$$\psi_{in}(a) = \psi_{out}(a)$$
$$Csin(\gamma a) = Ae^{-\kappa a}$$

Smoothness Condition:

$$\frac{\partial \psi_{in}(x)}{\partial x}|_{x=a} = \frac{\partial \psi_{out}(x)}{\partial x}|_{x=a}$$
$$\gamma Ccos(\gamma a) = -\kappa Ae^{-\kappa a}$$

Now to get rid of the constants and the exponential we will divide both equations by eachother. This will tell us when for what values of κ and γ there will be stationary states. The condition that tells us the quantization is

$$\gamma cot(\gamma a) = -\kappa \longrightarrow -\frac{\kappa}{\gamma} = cot(\gamma a)$$

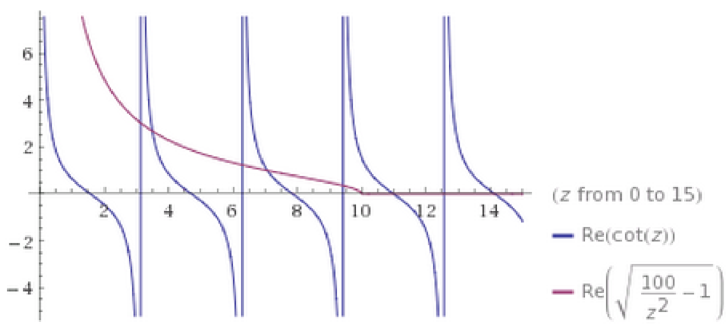
By squaring the separation constants (γ and κ) and adding them we get

$$\gamma^2 + \kappa^2 = \frac{2m(V_0 - E)}{\hbar^2} + \frac{2mE}{\hbar^2} = \frac{2mV_0}{\hbar^2} \longrightarrow \frac{\kappa}{\gamma} = \sqrt{\frac{2mV_0}{\gamma^2\hbar^2} - 1}$$

Now defining $z = \gamma a$ and $z_0 = \frac{a\sqrt{2mV_0}}{\hbar}$ we can rewrite the last expressions as

$$-\frac{\kappa}{\gamma} = -\sqrt{\frac{2mV_0}{\gamma^2\hbar^2} - 1} = cot(\gamma a) \longrightarrow \boxed{\sqrt{\frac{z_0^2}{z^2} - 1} = -cot(z)}$$

Here I graphed a very specific form of the boxed equation above. I made this plot using Wolfram Alpha.



2 Problem 2.38

A particle of mass m is in the ground state of the infinite square well. Suddenly the right wall expands to twice its original position-moving from a to $2a$- momentarily leaving the wavefunction undisturbed. The energy of the particle is now measured.

(a) What is the most probable result? What is the probability of getting that result?

(b) What is the next most probable result, and what is its probability?

(c) What is the expectation value?

2.1 Solution to Problem 2.38 Part A

We need to determine the wavefunction for all times t after the expansion. We do this using Fourier analysis. We know the wavefuntion of the infinite square-well of length $2a$. Hence,

$$\psi_n = \sqrt{\frac{2}{2a}} sin\left(\frac{n\pi}{2a}x\right) \qquad \text{With } E_n = \frac{n^2\hbar^2\pi^2}{8ma^2}$$

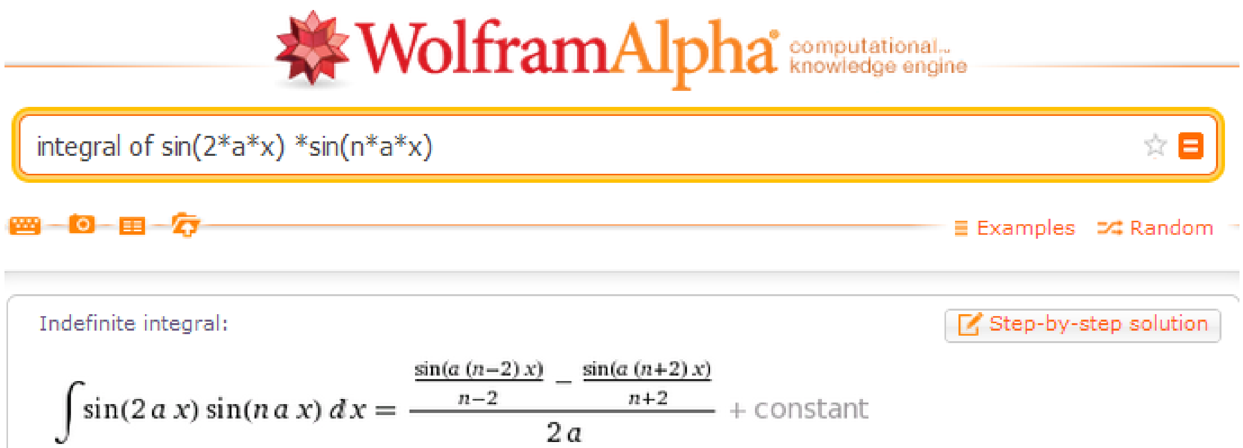
However, we also know that the initial wavefunction (any arbitrary function) in the infinite square well can be written as a sum due to completeness of the solutions.

$$\Psi(x,0) = \sum_n c_n\psi_n \longrightarrow c_n = \int \Psi(x,0)\psi_n dx$$

Where,

$$\Psi(x,0) = \sqrt{\frac{2}{a}} sin\left(\frac{\pi}{a}x\right)$$

Our job now is to determine the coefficients of our series representation of the initial wavefunction immediately after the wall has expanded. We will need to know one integral which I evaluated using Wolfram.



$$c_n = \int \sqrt{\frac{2}{a}}\sin\left(\frac{\pi}{a}x\right)\sqrt{\frac{2}{2a}}\sin\left(\frac{n\pi}{2a}x\right)dx$$

$$c_n = \frac{\sqrt{2}}{a}\int \sin\left(2\frac{\pi}{2a}x\right)\sin\left(\frac{n\pi}{2a}x\right)dx$$

$$c_n = \int \frac{\sqrt{2}}{a}\sin(2Ax)\sin(nAx)\,dx$$

$$c_n = \frac{\sqrt{2}}{a}\left.\frac{\frac{\sin(A(n-2)x)}{n-2} - \frac{\sin(A(n+2)x)}{n+2}}{2A}\right|_0^a$$

$$c_n = \frac{\sqrt{2}}{a}\left(\frac{\frac{\sin(\pi/2a(n-2)a)}{n-2} - \frac{\sin(\pi/2a(n+2)a)}{n+2}}{2A}\right) - (0)$$

$$c_n = \frac{\sqrt{2}}{a}\left(\frac{\frac{\sin(\pi/2(n-2))(n+2)-\sin(\pi/2(n+2))(n-2)}{n^2-4}}{\pi/a}\right)$$

We cannot evaluate when $n = 2$ since the solution in this form has a asymptotic behavior at $n = 2$. However, we see when n is even the argument of sine is an integer times π, which is zero. But when n is odd, the argument is an integer times $\pi/2$ which gives us a max value of one from the sines,

$$c_{odd} = \frac{\sqrt{2}}{a}\left(\frac{\frac{(n+2)-(n-2)}{n^2-4}}{\pi/a}\right) = \frac{\sqrt{2}}{a}\frac{4a}{\pi(n^2-4)} = \frac{4\sqrt{2}}{\pi(n^2-4)}$$

Now we must determine when $n = 2$. However, going back to our first form of our integral we find that

$$c_2 = \frac{\sqrt{2}}{a}\int \sin^2(\tfrac{\pi}{a}x)dx = \frac{\sqrt{2}}{a}\left[\frac{x}{2} - \frac{\sin(\frac{2\pi}{a}x)}{4\pi/a}\right|_0^a = \frac{\sqrt{2}}{a}\frac{a}{2} = \frac{1}{\sqrt{2}}$$

Therefore, we can write the coefficients as

$$c_n = \begin{cases} 0, & \text{for n even,} n \neq 2 \\ \frac{4\sqrt{2}}{\pi(n^2-4)}, & \text{for n odd} \\ \frac{1}{2}, & \text{n=2} \end{cases}$$

Now the probability of finding a particle in a certain state n is given by the square of coefficient c_n corresponding to that particular state. It is obvious that the most probable state is $n = 2$ with energy

$$\boxed{E_{n=2} = \frac{\pi^2\hbar^2}{2ma^2}}$$

and that state has probability

$$\boxed{P_2 = |c_{n=2}|^2 = \frac{1}{2}}$$

2.2 Solution to Problem 2.38 Part B

The next probable result is when n is odd. However, not just any odd state. It will either be when $n = 1$ or $n = 3$ due to the nature of the denominator. If $n = 3$ the denominator is $9 - 4 = 5$, and when $n = 1$ the denominator is $1 - 4 = -3$ which smaller in magnitude. Hence, the next probable state is $n = 1$. The energy is given as

$$\boxed{E_{n=1} = \frac{\pi^2\hbar^2}{8ma^2}}$$

with probability

$$\boxed{P_1 = |c_{n=1}|^2 = \frac{4^2(2)}{\pi^2(-3)^2} = \frac{32}{9\pi^2} \approx 0.36}$$

2.3 Solution to Problem 2.38 Part C

The expectation value of the energy is

$$\langle\psi|H|\psi\rangle = \int \psi^*\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\psi dx$$

Taking this definition of the expectation value of the energy we can see that the integral will be just a sine squared again (since $\psi \propto \sin(Ax)$)

$$\langle H\rangle = \frac{\hbar^2}{2m}\frac{2}{a}\int \sin(\tfrac{\pi}{a}x)(-\tfrac{-\pi^2}{a^2})\sin(\tfrac{\pi}{a}x)dx$$

$$\langle H\rangle = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}\frac{2}{a}\int \sin^2(\tfrac{\pi}{a}x)dx$$

$$\langle H\rangle = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}\frac{2}{a}\int \sin^2(\tfrac{\pi}{a}x)dx$$

$$\langle H\rangle = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}\frac{2}{a}\left[\tfrac{a}{2}\right]dx$$

$$\boxed{\langle H\rangle = \frac{\pi^2\hbar^2}{2ma^2}}$$

Please note that all of these integrals were evaluated over all space ($-\infty < x < \infty$) but since wavefunction only exist inside the well we evaluated each integral from 0 to a.

3 Problem 2.42

Find the allowed energies of the half harmonic oscillator

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2x^2, & \text{for } x > 0 \\ \infty, & \text{for } x \leq 0 \end{cases}$$

3.1 Solution to Problem 2.42

Since the potential is infinite at $x = 0$ the wavefunction must also be zero at $x = 0$. So we must analyze the solutions to the harmonic oscillator to and choose which ones can satisfy this new boundary condition. The solution to the harmonic oscillator is a Gaussian multiplied by a polynomial (specifically the Hermite polynomials). The solution is given by equation 2.85

$$\psi_n = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\xi) e^{-\xi^2/2}$$

Where $\xi \propto x$ and H_n are the Hermite polynomials and are given by table 2.1

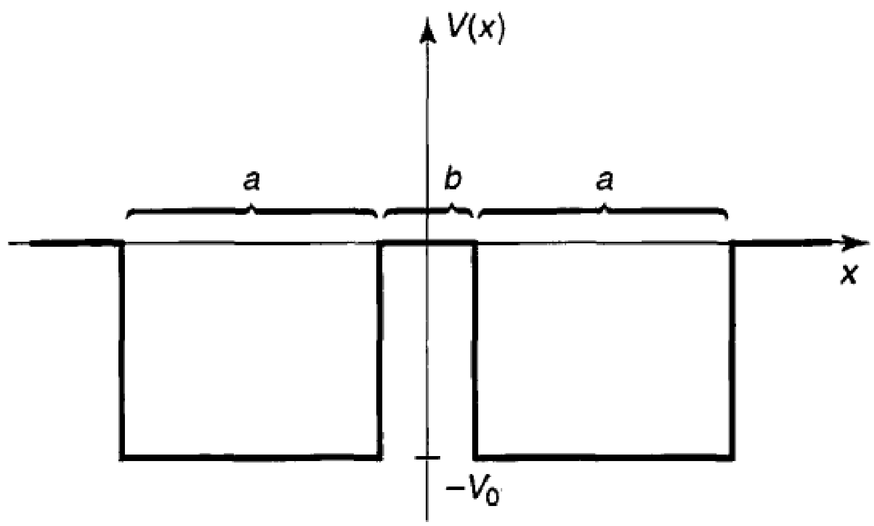
$$\begin{aligned}
H_0 &= 1 \\
H_1 &= 2\xi \\
H_2 &= 4\xi^2 - 2 \\
H_3 &= 8\xi^3 - 12\xi \\
H_4 &= 16\xi^4 - 48\xi^2 + 12 \\
H_5 &= 32\xi^5 - 160\xi^3 + 120\xi
\end{aligned}$$

From this we see a pattern that the odd states are all zero at $x = 0$ and all even states are non-zero at $x = 0$. Hence, the only states that can exist are the odd ones and the corresponding allowed energies are

$$\boxed{E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega \qquad \text{for odd n only}}$$

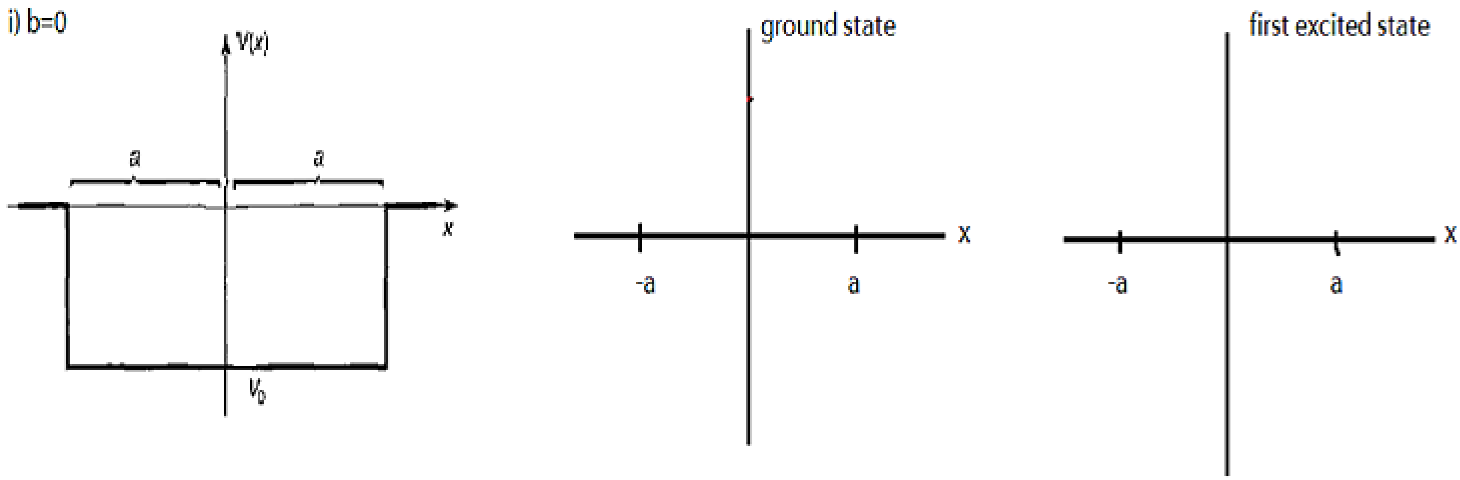
4 Problem 2.47

Consider the double square well where a and V_0 are large enough to contain several bound states.

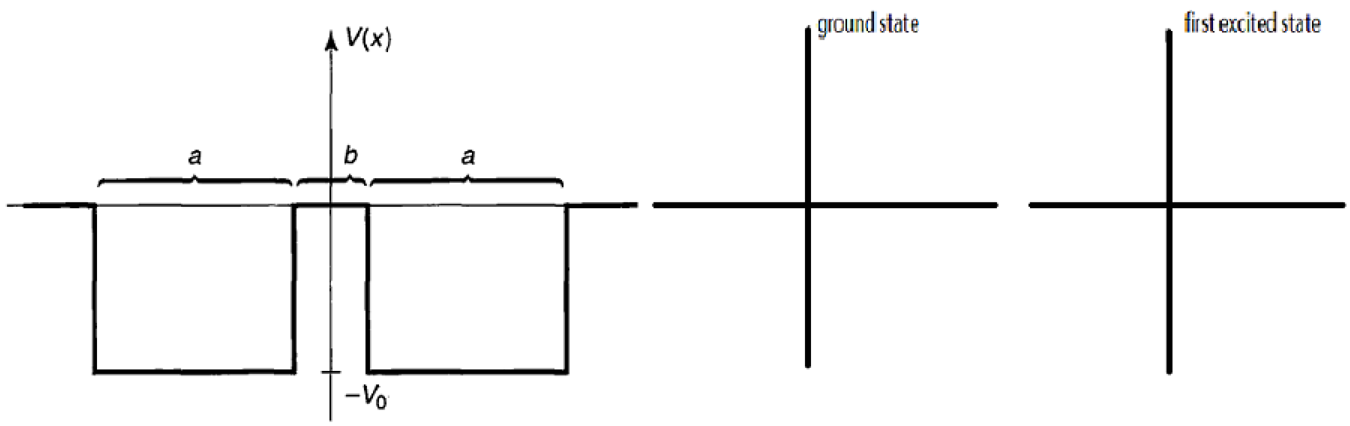


4.1 Solution to Problem 2.47 Part A

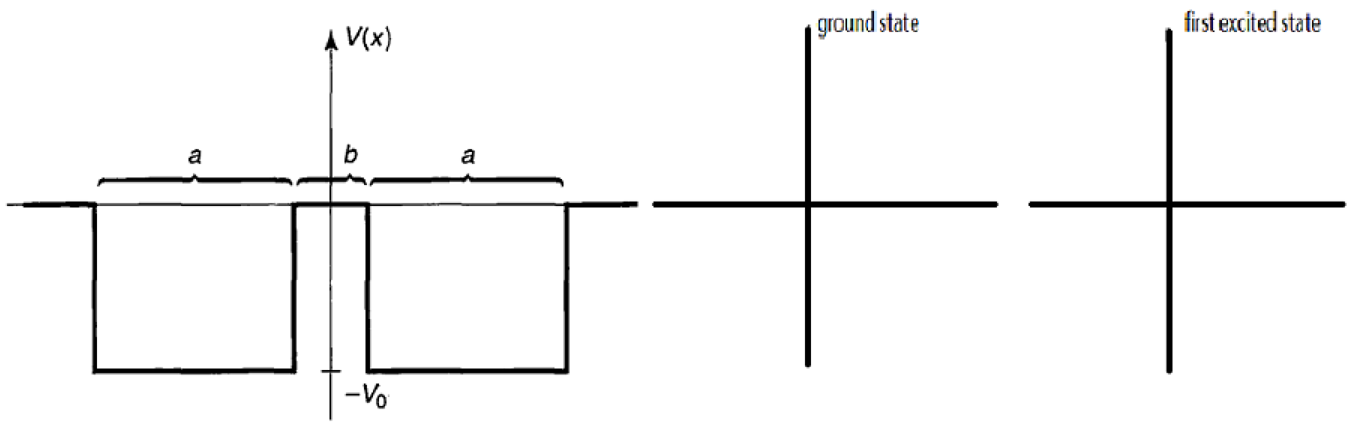
4.1.1 *i)* for $b = 0$



4.1.2 *ii)* for $b \approx a$



4.1.3 *iii)* for $b >> a$



4.2 Solution to Problem 2.47 Part B

As b goes from 0 to ∞ the wavefunction goes from a single finite square well to a double finite square well to two finite square well. All of these configurations we know the energy values of. A single finite square well energy is given by equation 2.157

$$E_n = V_0 + \frac{n^2\hbar^2\pi^2}{2ma^2}$$

and for two finite square wells ($b >> a$) it would just twice E_n, but since there is only one particle it should be in only one well hence the energy should approach the same value.

For the double finite well ($b \approx a$) we see that the wavefunction is actually amplitude is smaller for the first excited state (since it is an odd function) than the ground state (which is even). This means the ground state energy should increase and the first excited state should decrease, and eventually reach the same value.

4.3 Solution to Problem 2.47 Part C

Electrons want to be in low energy configuration. As we see from the graph the lowest energy configuration is when $b \longrightarrow 0$ for the ground state, which implies atoms/molecules would want to be in the ground state and as close as possible. However, for the first excited state the molecules have lowest energy when $b \longrightarrow \infty$, and hence would not want to come together.

5 Problem 3.35

Coherent states of the harmonic oscillator. Certain linear combination of stationary states can produce the limit in the uncertainty principle. These are the eigenstates of the lowering operator. Lets investigate them.

(a) Calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$ and $\langle p^2 \rangle$ for the state $|\alpha\rangle$.

(b) Calculate σ_x, σ_p, show that $\sigma_x \sigma_p = \hbar/2$

(c) Like any other wave function, a coherent state can be expanded in terms of ernergy eigenstates:

$$|\alpha\rangle = \sum_{n=0}^{\infty} c_n |n\rangle$$

Show that the expansion coefficients are

$$c_n = \frac{\alpha^n}{\sqrt{n!}} c_0$$

(d) Determine c_0 by normalizing $|\alpha\rangle$.

(e) Now put in time dependence and show that $|\alpha(t)\rangle$ remains an eigenstate of a_-, but evolves in time.

(f) Is the ground state itself coherent? If so, What is the eigenvalue?

5.1 Solution to Problem 3.35 Part A

Since we are dealing with coherent states, we know that $\langle x \rangle = \langle p \rangle = 0$ is not true. The trick now is to determine expectation values for the coherent state harmonic oscillator. We will use the technique in example 2.5 in Griffiths to evaluate these. We need the definitions of x and p in terms of the lowering and raising operators.

$$x = A(a_+ + a_-) \qquad p = iB(a_+ - a_-) \qquad \text{Where } A = \sqrt{\frac{\hbar}{2m\omega}} \quad \text{and} \quad B = \sqrt{\frac{\hbar m\omega}{2}}$$

The solutions that bring uncertainty to its limit are eigenfunctions of the lowering operator

$$a_-|\alpha\rangle = b|\alpha\rangle$$

5.1.1 Computing $\langle x \rangle$

Following example 2.5 we find that the expectation value of x is just

$$\begin{aligned}
\langle x \rangle &= \langle \alpha | x | \alpha \rangle \\
&= \langle \alpha | A(a_+ + a_-) | \alpha \rangle \\
&= A(\langle \alpha | a_+ | \alpha \rangle + \langle \alpha | a_- | \alpha \rangle) \\
&= A(\langle \alpha | (a_-)^* | \alpha \rangle + \langle \alpha | b | \alpha \rangle) \\
&= A(\langle \alpha | b^* | \alpha \rangle + b\langle \alpha | \alpha \rangle) \\
&= A(b^* \langle \alpha | \alpha \rangle + b\langle \alpha | \alpha \rangle) \\
&= \boxed{A(b^* + b) = \sqrt{\frac{\hbar}{2m\omega}}(b^* + b)}
\end{aligned} \tag{1}$$

5.1.2 Computing $\langle x^2 \rangle$

Following example 2.5 we find that the expectation value of x^2 is just

$$\begin{aligned}
\langle x^2 \rangle &= \langle \alpha | x^2 \alpha \rangle \\
&= \langle \alpha | (A^2 (a_+ + a_-)^2) \alpha \rangle \\
&= A^2 \langle \alpha | (a_+^2 + a_-^2 + a_+ a_- + a_- a_+) \alpha \rangle \\
&= A^2 \left(\langle \alpha | a_+^2 | \alpha \rangle + \langle \alpha | a_-^2 | \alpha \rangle + \langle \alpha | a_+ a_- | \alpha \rangle + \langle \alpha | a_- a_+ | \alpha \rangle \right) \\
&= A^2 \left(\langle \alpha | (a_-^2)^* | \alpha \rangle + \langle \alpha | a_-^2 | \alpha \rangle + \langle \alpha | a_+ a_- | \alpha \rangle + \langle \alpha | 1 + a_+ a_- | \alpha \rangle \right) \\
&= A^2 \left(\langle \alpha | (a_-^2)^* | \alpha \rangle + \langle \alpha | a_-^2 | \alpha \rangle + \langle \alpha | a_+ a_- | \alpha \rangle + \langle \alpha | 1 + a_+ a_- | \alpha \rangle \right) \\
&= A^2 \left((b^*)^2 \langle \alpha | \alpha \rangle + b^2 \langle \alpha | \alpha \rangle + 2 \langle \alpha | a_-^* a_- | \alpha \rangle + \langle \alpha | \alpha \rangle \right) \\
&= A^2 \left((b^*)^2 + b^2 + 2b^* b + 1 \right) \\
&= \frac{\hbar}{2m\omega} \left((b^*)^2 + b^2 + 2b^* b + 1 \right) \\
&= \boxed{\frac{\hbar}{2m\omega} \left((b^* + b)^2 + 1 \right)}
\end{aligned} \tag{2}$$

5.1.3 Computing $\langle p \rangle$

We will use the same technique as we did in computing $\langle p \rangle$

$$\begin{aligned}
\langle p \rangle &= \langle \alpha | iB(a_+ - a_-) | \alpha \rangle \\
&= iB \langle \alpha | a_+ - a_- | \alpha \rangle \\
&= iB(\langle \alpha | a_+ | \alpha \rangle - \langle \alpha | a_- | \alpha \rangle) \\
&= iB(\langle \alpha | a_-^* | \alpha \rangle - \langle \alpha | a_- | \alpha \rangle) \\
&= iB(b^* \langle \alpha | \alpha \rangle - b \langle \alpha | \alpha \rangle) \\
&= iB(b^* - b) \\
&= \boxed{\frac{2im\omega}{\hbar}(b^* - b)}
\end{aligned} \tag{3}$$

5.1.4 Computing $\langle p^2 \rangle$

$$\begin{aligned}
\langle p^2 \rangle &= \langle \alpha | p^2 \alpha \rangle \\
&= \langle \alpha | -B^2 (a_+ - a_-)^2) \alpha \rangle \\
&= -(\frac{1}{A^2} \langle \alpha | (a_+^2 + a_-^2 - a_+ a_- - a_- a_+) \alpha \rangle \\
&= -B^2 \left(\langle \alpha | a_+^2 | \alpha \rangle + \langle \alpha | a_-^2 | \alpha \rangle - \langle \alpha | a_+ a_- | \alpha \rangle - \langle \alpha | a_- a_+ | \alpha \rangle \right) \\
&= -B^2 \left(\langle \alpha | (a_-^2)^* | \alpha \rangle + \langle \alpha | a_-^2 | \alpha \rangle - \langle \alpha | a_+ a_- | \alpha \rangle - \langle \alpha | 1 + a_+ a_- | \alpha \rangle \right) \\
&= -B^2 \left(\langle \alpha | (a_-^2)^* | \alpha \rangle + \langle \alpha | a_-^2 | \alpha \rangle - 2 \langle \alpha | a_+ a_- | \alpha \rangle - \langle \alpha | \alpha \rangle \right) \\
&= -B^2 \left((b^*)^2 \langle \alpha | \alpha \rangle + b^2 \langle \alpha | \alpha \rangle - 2 \langle \alpha | a_-^* a_- | \alpha \rangle - \langle \alpha | \alpha \rangle \right) \\
&= -B^2 \left((b^*)^2 + b^2 - 2b^* b - 1 \right) \\
&= \frac{2m\omega}{\hbar} \left(2b^* b + 1 - (b^*)^2 - b^2 \right) \\
&= \boxed{\frac{2m\omega}{\hbar} \left(1 - (b^* - b)^2 \right)}
\end{aligned} \tag{4}$$

5.2 Solution to problem 3.35 Part B

First we must compute the standard deviation in position and momentum.

$$\begin{aligned}
\sigma_x &= \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \\
&= \sqrt{A^2 \left((b^*)^2 + b^2 + 2b^* b + 1 \right) - A^2 (b^* + b)^2} \\
&= A \sqrt{\cancel{(b^*)^2} + \cancel{b^2} + \cancel{2b^* b} + 1 - \cancel{(b^*)^2} - \cancel{b^2} - \cancel{2b^* b}} \\
&= \boxed{A}
\end{aligned} \tag{5}$$

Now the momentum

$$\begin{aligned}
\sigma_p &= \sqrt{\langle p^2 \rangle - \langle p \rangle^2} \\
&= \sqrt{-B^2\left((b^*)^2 + b^2 - 2b^*b - 1\right) + B^2(b^* - b)^2} \\
&= B\sqrt{-\cancel{(b^*)^2} - \cancel{b^2} + \cancel{2b^*b} + 1 + \cancel{(b^*)^2} + \cancel{b^2} - \cancel{2b^*b}} \\
&= \boxed{B}
\end{aligned} \tag{6}$$

Now the uncertainty relation gives us

$$\boxed{\sigma_x \sigma_p = AB = \sqrt{\frac{\hbar}{2m\omega}}\sqrt{\frac{\hbar m\omega}{2}} = \frac{\hbar}{2}}$$

5.3 Solution to Problem 3.35 Part C

We recognize that the state is given in the form of equation 3.45 and that c_n is determined from equation 3.46. We also use equation 2.66 and 2.67 to get general formula

$$\begin{aligned}
\sum_{n=0}^{\infty} c_n |n\rangle &= |\alpha\rangle \\
\langle \alpha_n | \sum_{n=0}^{\infty} c_n |n\rangle &= \langle \alpha_n | \alpha \rangle \\
\sum_{n=0}^{\infty} c_n \langle \alpha_n | n \rangle &= \frac{1}{\sqrt{n!}} (a_+^n) * \langle \alpha_0 | \alpha \rangle \\
c_n &= \frac{1}{\sqrt{n!}} (a_-^n) \langle \alpha_0 | \alpha \rangle \\
c_n &= \frac{1}{\sqrt{n!}} \langle \alpha_0 | (a_-^n) \alpha \rangle \\
c_n &= \frac{1}{\sqrt{n!}} b^n \langle \alpha_0 | \alpha \rangle \\
&\boxed{c_n = \frac{1}{\sqrt{n!}} b^n c_0}
\end{aligned}$$

Where we know that c_0 is defined as by equation 3.43 with $n = 0$ as $c_n = \langle \Psi_n | \alpha \rangle$, which is obviously just the Fourier "trick".

5.4 Solution to Problem 3.35 Part D

From equation 3.47 we see that the normalization condition is

$$1 = \sum_n |c_n|^2$$

Therefore, we can just use our result from part C in equation 3.47 and solve for c_0

$$\begin{aligned}
\sum_n |c_n|^2 &= 1 \\
\sum_n \frac{1}{n!} b^{2n} c_0^2 &= 1 \\
c_0^2 \sum_n \frac{(b^2)^n}{n!} &= 1 \\
c_0^2 e^{b^2} &= 1 \\
c_0^2 &= e^{-b^2} \\
&\boxed{c_0 = e^{-b^2/2}}
\end{aligned}$$

Where we have used the definition of the exponential as $e^x = \sum_0^{\infty} \frac{x^n}{n!}$

5.5 Solution to Problem 3.35 Part E

The time dependence is given by

$$|n\rangle \longrightarrow e^{-iE_n t/\hbar}|n\rangle$$

Plugging this into our definition of the state α and using our definition of the energy eigenvalues $E_n = \hbar\omega(n+1/2)$ we get

$$\begin{aligned}
|\alpha\rangle &= \sum_n c_n |n\rangle \\
|\alpha\rangle &= \sum_n c_n e^{-iE_n t}|n\rangle \\
|\alpha\rangle &= \sum_n c_n e^{-i\hbar\omega(n+1/2)t}|n\rangle \\
|\alpha\rangle &= \sum_n c_n e^{-i\hbar\omega t n} e^{-i\hbar\omega t/2)}|n\rangle
\end{aligned}$$

Now we use our definition of c_n from part C and D.

$$\begin{aligned}
|\alpha\rangle &= e^{-i\hbar\omega t/2)} \sum_n \frac{b^n}{\sqrt{n!}} e^{-b^2/2} e^{-i\hbar\omega n t}|n\rangle \\
|\alpha\rangle &= e^{-i\hbar\omega t/2)} \sum_n \frac{(be^{-i\hbar\omega t})^n}{\sqrt{n!}} e^{-b^2/2} e^{-i\hbar\omega n t}|n\rangle
\end{aligned}$$

From this we see that the time dependence just gives oscillatory motion (in time) to the original eigenfunction. Hence,

$$\boxed{b(t) = be^{-i\omega t}}$$

5.6 Solution to Problem 3.35 Part F

By definition of the lowering operator, it can operate on the ground state but it will produce zero. This is because there is lower limit on energy in quantum mechanics. Hence,

$$a_-|0\rangle = 0 \longrightarrow a_-|0\rangle = 0|0\rangle \longrightarrow b = 0$$