Low Frequency Impedances

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Abstract

By deriving the voltage divider equation from Kirchoff's Voltage Loop Law and Thevenin Equivalence we will determine the output impedance of three common laboratory electronic equipment. For elements that are frequency dependent we can utilize the idea of complex impedance and the properties of linear circuit elements to determine the components of a unknown electrical circuit. These components will consist of resistors, capacitors, and inductors in any combination. After we determine the components of four unknown circuits we will examine the properties of three non-linear circuit elements consisting of two diodes and a thermistor.

1 Introduction

The first step for determining output impedance of a circuit is the Thevenin voltage divider. First assume the circuit is a perfect voltage source in series with a resistance, where in this case the resistance is the output impedance. We then measure the open source voltage, or Thevenin voltage, with a voltmeter. Since it is an open circuit there is no current drawn and no voltage drop across the the internal resistance, Z, hence the voltage measured is the voltage of the ideal voltage source.

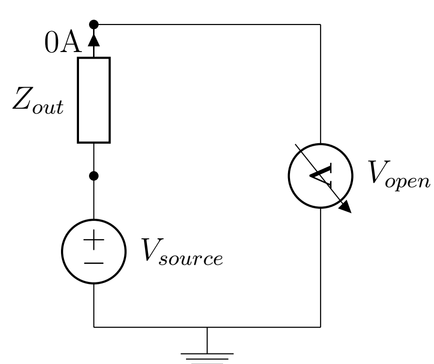

Figure 1: Open source voltage

Then we attach a known load[1] and measure the voltage across it. By measuring the voltage across a known resistor we can determine the current drawn being drawn by Ohm's Law, $I = \frac{V}{R}$. This is the same current passing through the internal output resistance.

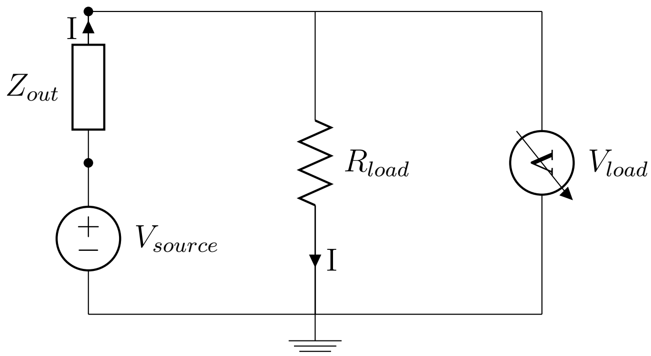

Figure 2: Determining Current

By using Kirchoff's Loop Law[2], $\oint_C \vec{E} \cdot d\vec{\ell} = \sum_{i=1}^{N} V_i = 0$, we can determine the internal resistance.

$$\Sigma V = V_{source} - IZ_{out} - V_{load} = 0 \tag{1}$$

Substituting $I = \frac{V_{load}}{R_{load}}$ and rearranging we get the voltage divider equation.

$$V_{load} = \frac{R_{load}}{Z_{out} + R_{load}} \cdot V_{source} \tag{2}$$

Rearranging again we obtain the equation for output impedance

$$Z_{out} = \frac{V_{load}}{V_{source} + V_{load}} \cdot R_{load} \tag{3}$$

A use of equation 3 can be seen pictorially in the following diagram.

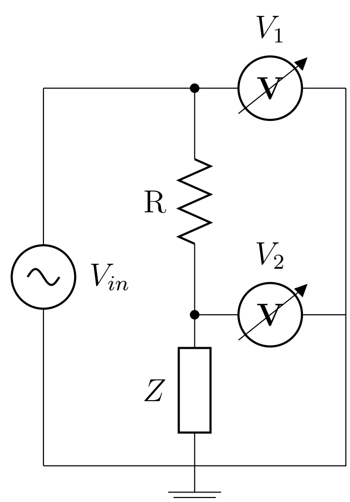

Figure 3: Voltage Divider of Box and known Resistance

[1]The load is just a circuit being powered, and in this case it is a resistor.

[2]Kirchoff's Loop Law states that the closed loop integral of the electric field in a circuit must be zero. Where the line integral of electric field is the electric potential.

Where the electrical potential difference between the measured voltages V_1 and V_2 is V_R and is defined as $V_R = V_1 - V_2$. This voltage is in phase with the current since the impedance of resistance is real, hence $I(t) = V(t)R_{load}$. Now, we have three techniques for discovering the components of an unknown circuit which are as follows; measure DC impedance directly with a ohmmeter, measure the attenuation of the voltage across the box at signal frequencies, and observing the phase difference between voltage across the box and current passing through it. please note that are systematic errors in all the following data due to the quality and precision of the equipment used. Also, note that all AC values will be presented in peak-peak form unless otherwise noted.

2 Output Impedance of Electronic Equipment

2.1 9-Volt Battery

Due to the nature of batteries it is logical that there would be some internal resistance. There are two types of batteries, ones that are producing the specified electromotive force and ones that are not producing the specified electromotive force. We shall look at both of these cases.

2.1.1 New Battery

We measured the open source voltage at $9.12V$, which is above the specified amount. We then used a substitution box as the load to obtain a voltage versus current graph. The voltage in the data table below refers to the voltage across the internal resistance which was deduced by using Kirchoffs Loop Law, $V = V_{internal} = V_{open} - V_{load}$. The current was deduced from knowing V_{load} and R_{load} and Ohms Relation, hence $I = \frac{V_{load}}{R_{load}}$.

Resistance(Ω)	Current (A)	Voltage (V)
100	0.081	1.02
90	0.0885	1.115
80	0.0952	1.504
70	0.1115	1.315
50	0.1477	1.735
30	0.218	2.48

Table 1: New Battery: $V_{open} = 9.12V$

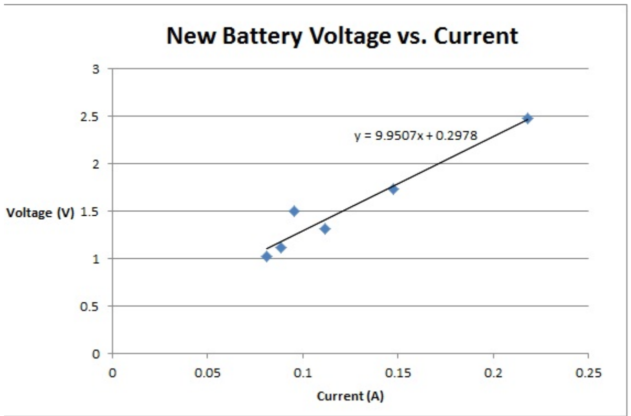

Figure 4: slope= $\frac{\Delta V}{\Delta I} = R_{new} = 9.95\Omega$

By applying Ohm's law we are able to determine the internal resistance of the new battery using the relation $R = \frac{\Delta V}{\Delta I}$. We were able to determine the resistance to be 9.95Ω. This is about what we would expect for a new battery since it is low enough to supply large current, $I = \frac{V_{open}}{Z_{out}} = \frac{9.12V}{9.95\Omega} = 0.92A$, to a large load ($R_{load} = 0\Omega$).

2.1.2 Old Battery

Resistance(Ω)	Current (A)	Voltage (V)
100	0.0615	1.68
90	0.0655	1.935
80	0.0731	1.982
70	0.08	2.23
50	0.0983	2.915
30	0.1352	3.3774

Table 2: Old Battery: $V_{open} = 7.83V$

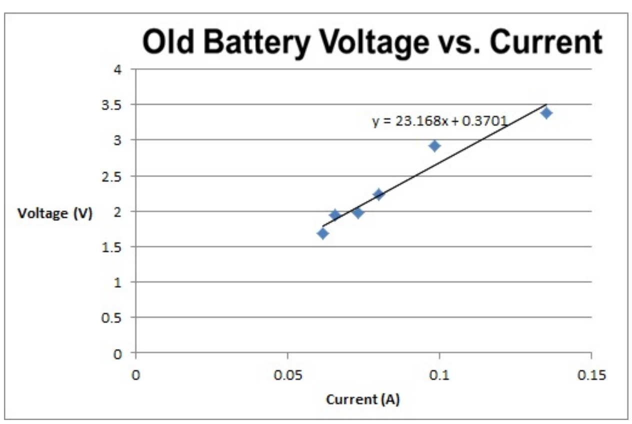

Figure 5: slope= $\frac{\Delta V}{\Delta I} = R_{old} = 23.15\Omega$

By applying Ohm's law again we are able to determine the internal resistance of the old battery to be 23.17Ω. This is about what we would expect for an old battery since it is high enough to supply only a small current, $I = \frac{V_{open}}{Z_{out}} = \frac{7.83V}{23.17\Omega} = 0.34A$, to a large load ($R_{load} = 0\Omega$).

By comparison we see that the new battery can supply significantly more power to a large load than the old battery. The new battery theoretically produces almost three times the amount of current than the old battery, $\frac{I_{new}}{I_{old}} = \frac{.92A}{.34A} = 2.71$. The new battery also supplies a larger electromotive force than the old battery, $\frac{V_{new}}{V_{old}} = \frac{9.12V}{7.83V} = 1.16$. Since power,$P$, is defined as the voltage times the current, $P = VI$, we can can determine the ratio of the power output of the new and old battery. The ratio is then just the current ratio multiplied by the voltage ratio $\frac{P_{new}}{P_{old}} = \frac{9.12V \cdot 0.92A}{7.83V \cdot 0.34A} = 2.71 \cdot 1.16 = 3.65$. The power output of the new battery is more than three times as large for a 9.12V new battery compared to a 7.83V old battery. We also see that the output impedance of the old battery is almost two and a half times larger than the output impedance of the new battery, $\frac{Z_{OLD}}{Z_{NEW}} = \frac{23.17\Omega}{9.95\Omega} = 2.33$.

2.2 Tenma Audio Generator 72-455A

The Tenma Audio Generator 72-455A is a complicated electronic circuit consisting of non-active and active circuit elements such as transistors and diodes. However, the Thevinin and Kirchoff approach to determining output impedance is still applicable to this complicated electronic device. According the data sheet provided by the manufacturer the output impedance should 600Ω. To determine the output impedance we used equations 1 through 3, and the experimental setup portrayed in figure 3.

Resistance(Ω)	Current (A)	Voltage (V)
100	0.01882	11.488
400	0.01323	8.078
500	0.01204	7.35
600	0.01104	6.746
700	0.0102	6.23
800	0.00947	5.794
900	0.00885	5.405
1000	0.0083	5.07

Table 3: Signal Generator: $V_{open} = 13.37V_{RMS}$

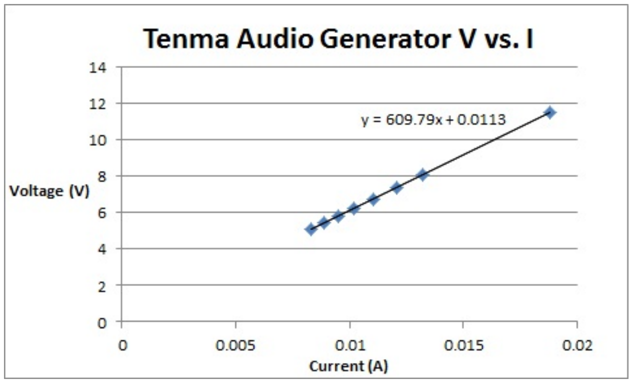

Figure 6: slope= $\frac{\Delta V}{\Delta I} = R_{Tenma} = 609.8\Omega$

We see a linear relationship between voltage and current resulting in a slope of 609.8 Ω which is what we expect since the generator is rated for 600Ω output impedance as specified above. The plot of voltage versus current shows us a slope, or resistance according to Ohm's law, of 609.8Ω. This is within 2% accuracy of the manufacturers data sheet specifications just as we should expect. Although far from an ideal signal generator it makes up for the large output impedance by low distortion and large slew rate.

2.3 Lambda LPD 422A-FM Regulated Power Supply

This voltage source offers two settings: current limiting and voltage regulating. We will measure the output impedance of both settings.

2.3.1 Current Limiting

We observed current limiting behavior at 13.36mA. Just before the current stopped increasing we saw a significant drop in voltage from almost $5V$ to $1V$. When the load was increased below 50Ω ($R_{load} < 50$ Ω) the power output dropped significantly. Specifically, the voltage continued to drop and the current stayed at a constant 13.36mA, just as expected.

Resistance(Ω)	Current (A)	Voltage (V)
100000	0.00005	5
50000	0.0001	5
10000	0.00049	4.9
5000	0.00099	4.95
1000	0.00495	4.95
500	0.00983	4.915
100	0.01017	1.017
50	0.01384	0.692

Table 4: Current Limiting: $V_{open} = 5V$

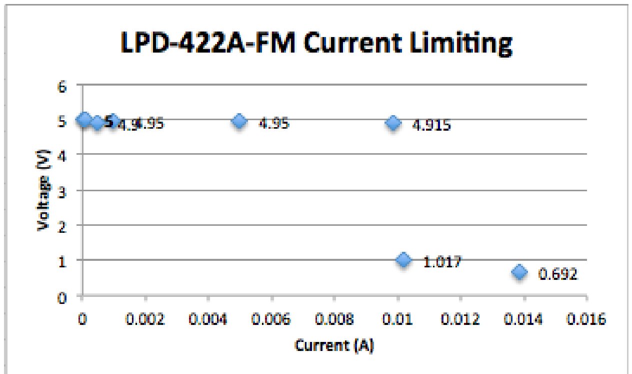

Figure 7: Current limiting effects

As the graph above shows, the internal resistance would be negative indicating improper operation for the current limiter. We know that the current limiter should behave this way since its job is to limit the current without varying the voltage significantly. However, it appears the current gets limited by dropping the voltage to match the desired current limit.

2.3.2 Voltage Regulating

We started with a small load of about $100k\Omega$ and increaseed it until we got to a large load , $R_{load} \approx 50\Omega$. We saw no voltage change by doubling the load from $100k\Omega$ to $50k\Omega$, however, increasing the load by a factor of 10 dropped the output voltage by 2% to $4.9V$. By the time we had increased the load to the maximum value of 50Ω, the output voltage had dropped by 12.1% to a value of $4.46V$.

Resistance(Ω)	Current (A)	Voltage (V)
100000	0.00005	5
50000	0.0001	5
10000	0.00049	4.9
5000	0.00099	4.95
1000	0.00495	4.95
500	0.00984	4.92
100	0.047	4.7
50	0.0892	4.46

Table 5: Voltage Regulating: $V_{open} = 5V$

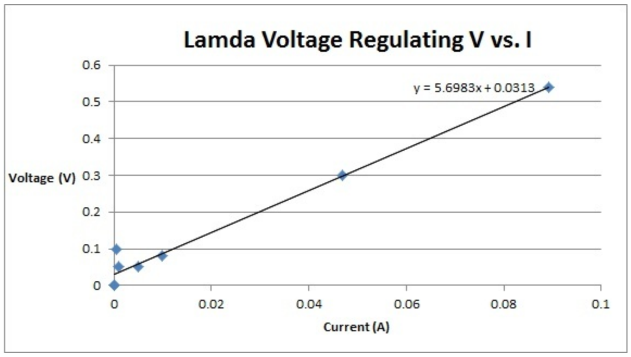

Figure 8: slope= $\frac{\Delta V}{\Delta I} = R_{LamdaVR}$

The graph above shows the slope, or resistance, of the V vs. I graph at about 5.7Ω. This is what we would expect for a decent voltage source, although ideally it would be ≈ 0Ω.

3 Impedances

3.1 Box A

We used the experimental setup from the previous section (Refer to figure 3 and equations 1 through 3 for details). First we attempted to measured the DC impedance directly with an ohmmeter but found it was too large for our ohmmeter. This could imply a very large resistance or a capacitor in series with other components. To determine whether it was capacitor or not we used the voltage divider method of figure 3. If it was a capacitor the voltage drop across the box would decrease as we increased the input AC voltage frequency, and the current through it would lead the voltage across it because of the nature of the complex impedance, $Z_C = \frac{1}{j\omega C}$. When we increased the frequency we observed a steady decline in voltage across the box, and hence a steady decline in the impedance. As we increased our frequency to $100kHz$ we saw no indication of a resonance so therefore we concluded there was no inductor.

This implied that it was either only a capacitor or a capacitor in series with a resistor. We knew that as we increased our frequency past about 100kHz we would begin seeing a parasitic capacitance. However, parasitic capacitance tends to be very small (on the pico-Farads scale), so it would have a very large impedance compared to our much larger capacitance. This observation could determine if there was actually a resistance in series with the box. As we table 6 shows at almost 1MHz the effective resistance dropped to about 30Ω, which implies that there is no resistor or one that is less than 30 Ω. We decided that the possible resistor was due to slight impedance of the capacitor.

Frequency	V_1	V_2	ΔV	$R_{effective}$	$C_{effective}$
100.2	4.84	4.8	0.04	240000	6.61823E-09
992.1	4.84	4.8	0.04	240000	6.68427E-10
9300	4.84	3.6	1.24	5806.451613	2.94732E-09
94000	4.84	0.55	4.29	256.4102564	6.60324E-09
14200	4.84	2.72	2.12	2566.037736	4.36786E-09
19500	4.84	2.16	2.68	1611.940299	5.06334E-09
25200	4.84	1.76	3.08	1142.857143	5.52622E-09
29500	4.84	1.56	3.28	951.2195122	5.67176E-09
34500	4.84	1.36	3.48	781.6091954	5.90217E-09
970000	4.84	0.072	4.768	30.20134228	5.43279E-09

Table 6: Box A: Capacitance

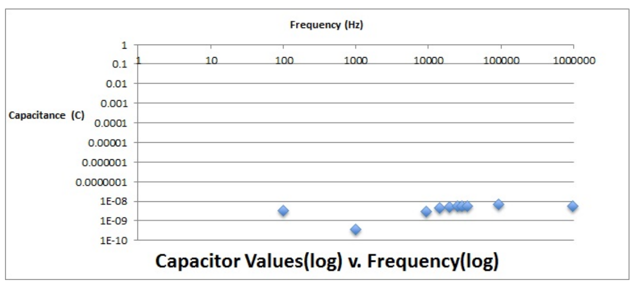

Figure 9: Average capacitance = $5.5n$F = $5500p$F

Starting with the complex impedance of a capacitor and using Ohms relation we obtained

$$Z_C = \frac{1}{2\pi fC} = \frac{V_2}{I} = \frac{V_2}{\frac{V_1 - V_2}{R_{load}}} = \frac{V_2 R_{load}}{V_1 - V_2} \tag{4}$$

Rearranging we obtain

$$C_{effective} = \frac{V_1 - V_2}{2\pi f V_2 R_{load}}$$

Where V_2 and f are variables. Averaging the capacitance values we found that the capacitor is about $5.5nF$. This is within a standard value range for a capacitor.

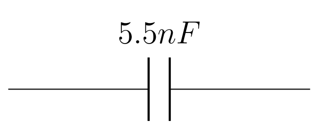

Figure 10: Predicted Circuit for Box A

3.2 Box C

We started with a DC impedance measurement using an ohmmeter and found it to be approximately 110.2Ω, which we shall define as R_{box}. This implied it was an inductor and/or resistor or an inductor and/or resistor in parallel with a capacitor. To determine if it was some combination of an LR or LRC circuit we swept the frequency and concluded that there was a resonance at around 18.8kHz. We then calculated the complex impedance of our predicted circuit

$$Z = R_{box} + \left(\frac{1}{2\pi f L} + 2\pi f C \right) \tag{5}$$

Where R_{box} is the resistance inside the box which is in series with the parallel combination of the inductor, L, and capacitor, C. Since there are three unknowns we took advantage of the resonant frequency relation, $\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$, and solved for L [1]

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \longrightarrow L = \frac{(2\pi f)^2}{C} = \frac{1}{AC} \tag{6}$$

Where we defined $A = \frac{1}{(2\pi f)^2}$. Combining the equations 5 and 6 and rearranging we find our capacitor value to be

$$C = [4\pi f_0 (Z - R_{box})] = 360x10^{-12}F$$

Where $Z = \frac{V_2}{I}$, $f_0 = 18,800Hz$, and R_{box} is the measured 110.2Ω. we know we can determine the capacitor value at the resonant frequency which was determined to be $360pF$. Now that we know the capacitor value we can refer back to the resonant frequency relation

$$LC = \frac{1}{(2\pi f_0)^2} = \frac{1}{(2\pi \cdot 18,800s^{-1})^2} = 7.17x10^{-11}s^2$$

Solving for L we found that the inductor value was $200mH$.

Frequency (Hz)	V_1 (V)	V_2 (V)	$R_{effective}$ (Ω)
9900	2.6	0.952	1155.33981
12100	2.6	1.3	2000
14200	2.64	1.72	3739.13043
16200	2.64	2.12	8153.84615
19300	2.64	2.28	12666.6667
23000	2.64	2.04	6800
27100	2.6	1.6	3200
35200	2.64	1.16	1567.56757
18800	2.64	2.26	11894.7368

Table 7: Box C

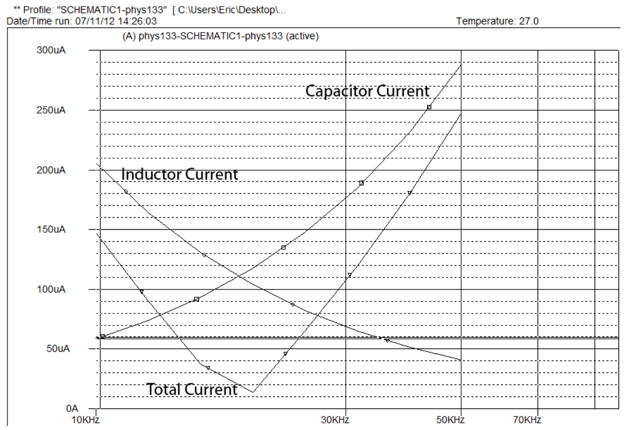

Figure 11: Predicted Circuit I vs. f: Resonance Frequency at 18.764 kHZ

We built our predicted circuit in the Orcad program called Capture and tested it in the Orcad program called Pspice and we found that the resonance of the simulation (Results are in figure below) agreed almost exactly with what we observed, a simulated resonance at 18.764 kHz and observed and predicted resonance at 18.8kHz.

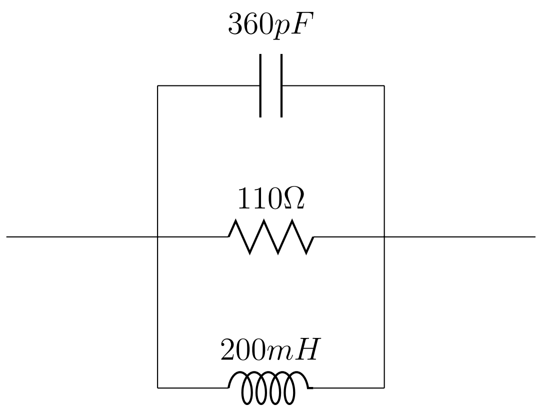

Figure 12: Predicted Circuit for Box C: Parallel LRC circuit

However, since any real inductor can be modeled as a perfect inductor in series with a resistor we can model it as the following

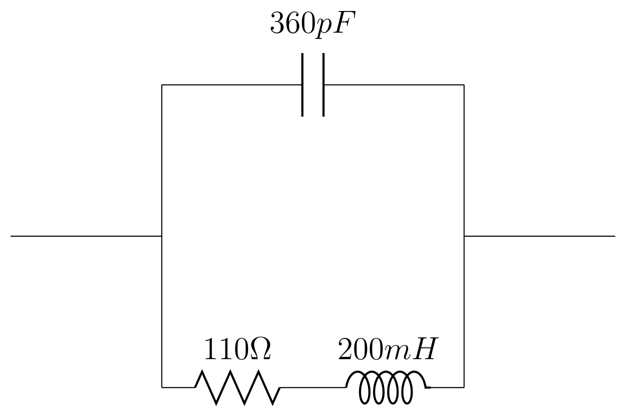

Figure 13: Modeled Circuit for Box C: Parallel LRC circuit

We ran both configurations of the possible parallel in a Pspice simulation and found that both gave a resonance at around 18.8kHz. This indicates that our model was in fact a good representation of realistic inductor (i.e non-zero resistance) in parallel with a capacitor and resistor. This independent relation of the resistance is what we would expect for a resonant frequency of given independent of the resistance, $\omega_0 = \frac{1}{\sqrt{LC}}$ [1].

3.3 Box D

We began with a direct measurement of the DC impedance with a ohmmeter and found that it was too large for the ohmmeter to determine. This indicates that we have either a very large resistor or a capacitor in series with other components. As we increased the frequency we observed a continual decrease in the voltage drop across the box, or similarly a decrease in impedance. There was no evidence of resonance as we varied the frequency, even as we got up into the parasitic capacitance frequencies.

Frequency (Hz)	V_1	V_2	$R_{effective}$	$C_{effective}$
10.6	2.62	2.6	260000	5.7772E-08
93.28	2.62	2.6	260000	6.565E-09
1040	2.62	2.08	7703.7037	2.0137E-08
9800	2.62	0.48	448.598131	4.7128E-08
7500	2.62	0.58	568.627451	4.5672E-08
5400	2.62	0.76	817.204301	4.1325E-08
3900	2.62	1	1234.5679	3.6096E-08
2500	2.62	1.36	2158.73016	3.0983E-08
12500	2.62	0.32	278.26087	7.3065E-08
45000	2.62	0.146	118.027486	2.5213E-07
831000	2.62	0.13	104.417671	4.5855E-07
1010000	2.62	0.13	104.417671	3.7728E-07

Table 8: Box D

At about 1MHz we saw that the effective impedance of the box has leveled to a constant

104Ω, which we will refer to as R_{box}. We decided that the parasitic capacitance would not alter the results significantly since the circuit would look like a a tiny capacitor in parallel with a large capacitor and small resistor as depicted in the figure below.

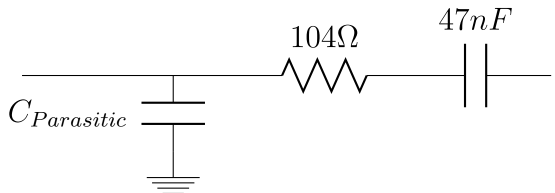

Figure 14: Predicted Circuit for Box D with parasitic capacitance

Assuming the parasitic capacitance has a negligible effect ($C_{parasitic} << C_{actual}$) we can write the impedance in complex form as $Z = R_{box} + \frac{1}{2\pi fC}$ and equate it to the Ohms relation definition, $Z = \frac{V}{I}$ (Refer to equation 4 for derivation of the following equation).

$$Z = \frac{V_2 R}{V_1 - V_2} = R_{box} + \frac{1}{2\pi fC}$$

Solving for C we get

$$C = \frac{1}{(Z - R_{box}) \cdot 2\pi f}$$

Where Z and f are variables and $R_{box} = 104\Omega$. The C value we found above corresponds to the $C_{effective}$ in the table above. We determined the capacitor value to be $0.047\mu F$, which is a standard capacitor value.

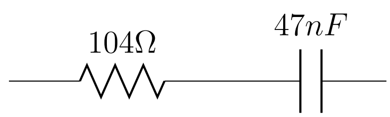

Figure 15: Predicted Circuit for Box D

3.4 Box G

We measured the DC impedance directly with ohmmeter to be approximately 0.02Ω indicating either a wire or a perfect inductor. Since realistically all inductors have some DC impedance at ≈ 50 Ω we assumed it was a wire. Just to be sure we used the setup portrayed in figure 3 to measure the voltage and current. The voltage across the box stayed at a constant ≈ $0V$ and current through the box stayed at a constant ≈ $2.45mA$ (peak-peak) as we swept the frequency range with a known resistance of 2kΩ.

Frequency (Hz)	V_1 (V)	V_2 (V)	$R_{effective}$ (Ω)
0	5	0	0.02
10	5	0	0.02
100	5	0	0.02
1000	5	0	0.02
10000	5	0	0.02
100000	5	0	0.02

Table 9: Box G: Wire

There was no need for further investigation to determine that it was indeed a wire.

4 Non-Linear Elements

4.1 Box J: Diode

By using the digital oscilloscopes XY mode we were able to compare current to voltage of the box by direct measurement and by using Ohm's relation. We noted a very steep exponential curve characteristic of the Ebers-Moll equation,

$$I = I_S \left(e^{\frac{eV}{kT}} - 1 \right) \tag{7}$$

Where I_S is the constant reverse saturation current and is dependent on the properties of the semiconducting material from which the p-n junction is made. By using the same approach as previously used in section two (Refer to figure 3).

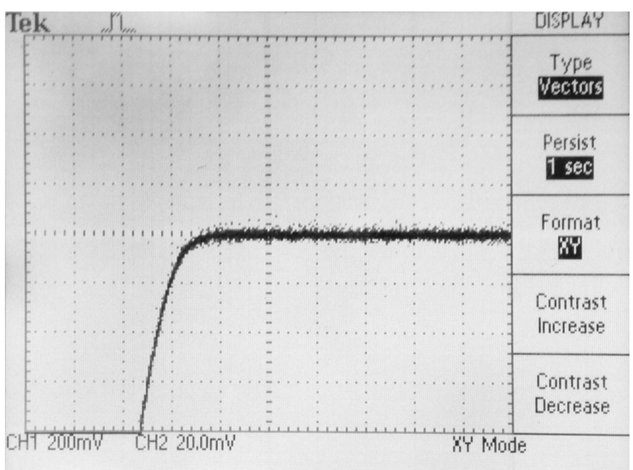

Figure 16: Characteristic Diode Curve. Note that positive current is downward and positive voltage is leftward.

In the picture above taken of the oscilloscope we observed the characteristic turn on forward biased voltage of about $0.6V$ and the steep exponential increase of current with only a tiny change in voltage.

4.2 Box K: Zener Diode

We observed the current and voltage relationship of the Zener diode. Using an apparatus similar to Figure 3 allowed us to measure the current through the element by measuring the voltage across a known resistor value ($2k\Omega$). The voltage across this resistor will be proportional to the current flowing through it. The sinusoidal wave generator set near $100Hz$ with a $6V$ amplitude allows us to vary the voltage across the acceptance range of the diode. The oscilloscope's mode was set to allow observing the voltage in X and current in Y.

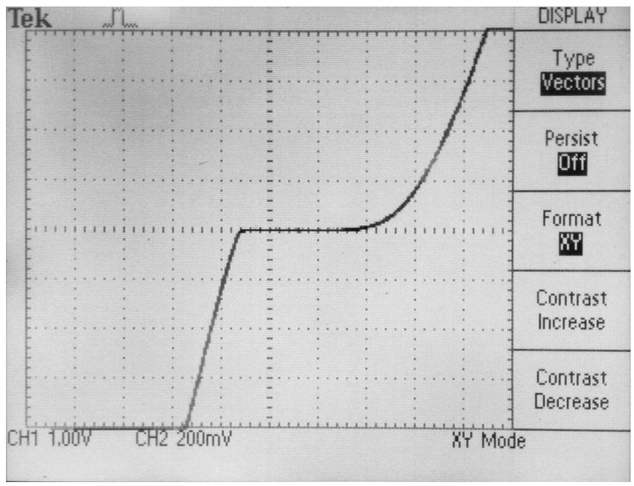

Figure 17: Characteristic Zener Diode Curve. Please note that this curve should be inverted.

We observed a breakdown voltage near $-2V$ with current rapidly increasing with decreasing voltage, seen on the right of figure above (Note: The graph is positive in the third quadrant). The diode "turns on" near the expected $0.6V$ when being forward biased. This follows our qualitative expectations for a Zener diode of breakdown voltage of $-2V$. However, we were expecting a harder shoulder for the Zener breakdown voltage. The "turned on" shoulder is typically harder than the "turned off" (Zener voltage) shoulder.

4.3 Box I: Thermistor

A thermistor's resistance varies exponentially with changing temperature. Our experimental setup was similar to the setup of section 4.2 (Refer to figure 3). We determined the current flowing through the thermistor by measuring the voltage across the known resistor using Ohm's Relation, $V = IR$. The oscilloscope was set in XY mode, such that channel one was X or voltage across box and channel two was Y or voltage across resistor that is proportional current. We heated the thermistor by continuously driving current through it. By setting the power supply at a DC value near the safe limit (around $10V$) we heated the thermistor. We expect from theory that the element's resistance will decrease with temperature increase [1].

$$R = Ae^{\frac{B}{T}}$$

Where A is a normalizing constant for the initial temperature and B is a parameter derived from the Steinhart-Hart equation. Initially we took a thermistor at room temperature that had not been heated up. We rapidly changed the voltage from $0 - 10V$. We observed that the thermistor behaved like a resistor across the range of voltages, indicating that the theory for low temperature is correct. The thermistor's temperature presumably did not change dramatically as the curve drawn on the oscilloscope was strongly linear and held a constant slope of approximately $\frac{1}{3}$.

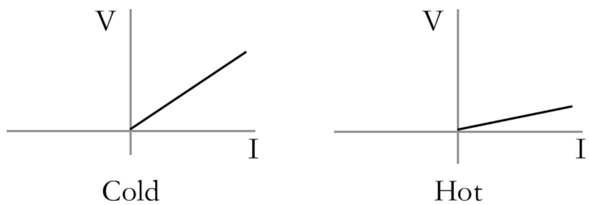

Figure 18: I vs. V for Low Temperature and High Temperature.

Secondly, we heated the element by driving current through it by increasing the power supplies voltage to around 12V for approximately fifteen seconds. Rapidly changing the voltage from the power supply we observed an I-V curve with a smaller slope. Heating up the element caused the slope to reduce by more than one-third of its value from the initial case to a value of approximately $\frac{1}{10}$.

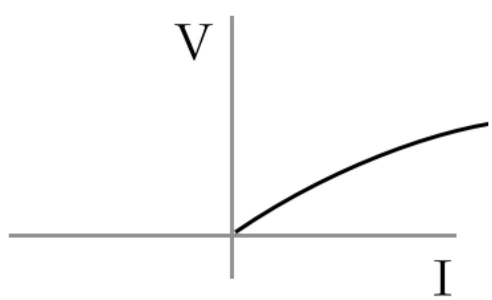

Figure 19: I vs. V for gradual heating

Finally, we allowed the element to cool then observed the I-V while very gradually changing the power supply's voltage. We observed a curve with decreasing slope as voltage increased, or that resistance was decreasing with increasing temperature.

References

[1] George Brown, Fred Kuttner. Physics 133 Lab Manual. Summer 2012.