

# Math 23A Practice Midterm 2

Eric Reichwein  
Department of Physics  
University of California, Santa Cruz

November 19, 2013

## Problem 1

### Problem 1 Part A

Let  $\vec{r}(t) = (x(t), y(t))$  be a parametrization for the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$  with  $\vec{r}(0) = (2, 0)$ . Let  $f(x, y) = x^3 + xy + y^3$  and  $\vec{R}(t) = (x(t), y(t), f \circ \vec{r}(t))$ . Find the equation for the tangent line to  $\vec{R}(t)$  at  $t = 0$ .

### Problem 1 Part B

Let  $f(x, y) = (2 + xy^2, x^2 + y^2, 3 + y^3)$  and  $T(r, \theta) = (r \cos \theta, r \sin \theta)$ . Compute  $D(f \circ T)(2, \frac{\pi}{2})$ .

### Problem 1 Part C

Find the equation for the line that is normal to the graph  $f(x, y) = y^2 - x^3$  at  $(1, 3, 8)$

## Problem 2

You are walking on a surface given by  $Q(x, y) = y \sin(\pi x) - x \sin(\pi y) + 10$

### Problem 2 Part A

From the point  $(2, 1, 10)$ , what  $x, y$ -direction should you follow to go up the fastest ?

### Problem 2 Part B

Find an  $x, y$ -direction from  $(2, 1, 10)$  that will keep you at the same level.

### Problem 2 Part B

What slope will you encounter as you set off from  $(2, 1, 10)$  towards  $(-1, 7, 10)$ ?

### Problem 3

#### Problem 3 Part A

What is the rate of change of  $f(x, y, z) = e^x \cos z - y$  at  $(1,1,1)$  in a direction normal to  $x^2z + z^y = 2$  at  $(1,1,1)$ ?

#### Problem 3 Part B

Find the second order Taylor polynomial for  $f(x, y) = y^2e^{-x^2}$

## Problem 4

Find and classify all critical points for the function  $G(x, y) = \frac{1}{4}x^4 - \frac{5}{3}x^3 + y^3 + 3x^2 - \frac{3}{2}y^2 + 20$