Math 23A Practice Midterm 1

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October 21, 2013

1 Problem 1

1.1 Problem 1 Part A

Find a vector that has length 3 and makes an angle of $\frac{\pi}{3}$ with $\vec{k} = (0, 0, 1)$.

1.2 Problem 1 Part B

Find the equation to the line that is parallel to the plane $x + y + z - 4 = 0$ and passes through (1,1,1).

1.3 Problem 1 Part C

Find the equation for the line that is normal to the graph $f(x, y) = y^2 - x^3$ at (1,3,8)

1.4 Problem 1 Part D

If $f(x, y, z) = xe^{z-y}$ find

$$\lim_{h \to 0} \frac{f(1 + h, 1, 2) - f(1, 1, 2)}{h}$$

2 Problem 2

Let $\vec{v} = (2, 2, 0), \vec{w} = (0, 3, 1), \vec{u} = (1, 1, 4)$

2.1 Problem 2 Part A

Draw the resulting parallelpiped. And determine the distance from the base (containing $\vec{v}, \vec{w}$) to the top.

2.2 Problem 2 Part B

Find the volume of the the parallelpiped and its surface area.

3 Problem 3

Problem 3 Part A

Find the equation for the tangent plane to $f(x, y) = ye^{-x^2-y^2}$ at (3,2).

Problem 3 Part B

At what (x, y) point will $f(x, y) = x^3 - y$ have a tangent plane parallel to the plane found in part b?

4 Problem 4

Consider the function

$$f(x,y)f(n) = \begin{cases} \frac{xy^4}{x^2+y^8}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if if } (x,y) = (0,0) \end{cases}$$

Problem 4 Part A

Show, using defintion of partial derivatives, that both $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at (0,0).

Problem 4 Part B

Show that f is not differentiable at (0,0) by showing the limit as $(x,y) \longrightarrow (0,0)$ does not exist.

5 Problem 5

5.1 Problem 5 Part A

If $\vec{r}(t) = (t\sin t, t^3, t\cos t, e^t)$ compute $\vec{r}'(0)$.

5.2 Problem 5 Part B

If $g(x, y, z, w) = (xy, -yz, zw, -wx)$ compute $Dg(1, 2, 3, 4)$.