Physics 6C Review 2

Eric Reichwein
Department of Physics
University of California, Santa Cruz

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Contents

1 Capacitors

1.1 Capacitance

In the 18th century an area of research that was of great interest was storing energy that could later be used to do work. We know that when two charges are in a system together and are separated by some distance there is electrical potential energy in that system. These charges then have the potential to do work. This is the main idea of capacitors. The separation of oppositely charged particles creates a potential difference, where the energy is actually stored in the electric field. The capacitor does exactly that and we can see how in the parallel plate capacitor.

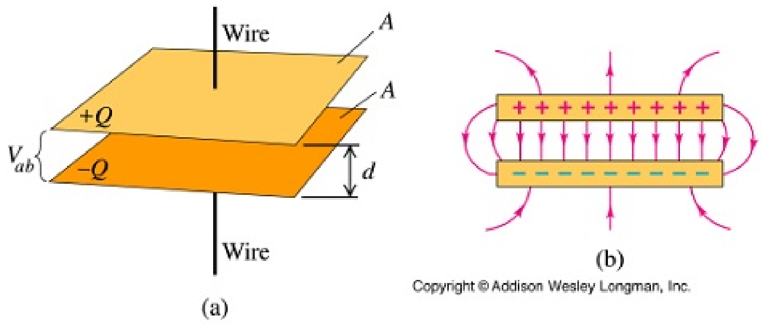

Figure 1: (a) Parallel plate capacitor geometry (b) the electric field produced by capacitors

First remember what the electric field is above a conducting plane, $\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{z}$, as derived using Gauss' Law. However, since there is one plate with $-\sigma$ and one with $+\sigma$ surface charge densities, one electric field points inward and one puts outward from the plate, hence they both contribute to the total electric field

$$\vec{E}_{total} = \frac{\sigma}{2\epsilon_0}\hat{z} + \frac{\sigma}{2\epsilon_0}\hat{z} = \frac{\sigma}{\epsilon_0}\hat{z}$$

Since we want to find the energy stored in the electric field in the capacitor we will integrate the electric field along the distance between the plates.

$$V = V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell} = -\int_a^b \frac{\sigma}{\epsilon_0} d\ell cos(180°) = \frac{\sigma \cdot d}{\epsilon_0}$$

Where d is the distance between the plate a and plate b, and the $d\ell cos(180°)$ comes from the dot product $\vec{E}$ and the infinitesimal length $d\vec{\ell}$[1] By using the definition of surface charge density, $\sigma = \frac{Q}{A}$, we can simply the electric potential energy equation even more.

$$V = \frac{Q \cdot d}{A\epsilon_0}$$

Now we have a relation between the electric potential energy and the total charge which is entirely dependent on the geometry of the separated conductors. This quantity is what we define as capacitance

$$\frac{Q}{V} = \frac{A}{d}\epsilon_0 = C$$

[1]The $d\vec{\ell}$ the differential form of the unit vector $\hat{z}$ seen in the previous equation for the electric field of the conductor plane.

1.2 The energy in a Capacitor

Using the definition of work, $W = U_{ab} = Vq$, and by differentiating both sides we obtain the differential form, $dW = V dq$. Where now we can determine the infinitesimal amount of work done by adding an infinitesimal amount charge to the capacitor and then integrate both sides as follows

$$\int_0^W dW = \int_0^Q V dq$$

Where now we can use the definition of capacitance, the ratio of charge to voltage, $V = \frac{q}{C}$, since we are integrating over charge not voltage.

$$\begin{aligned} \int_0^W dW &= \int_0^Q V dq \\ W - 0 &= \int_0^Q \frac{q}{C} dq \\ W &= \left. \frac{q^2}{2C} \right|_0^Q \\ W &= \frac{Q^2}{2C} \end{aligned}$$

Where $W = U_{ab}$ which is the energy stored in the capacitor. We can also $Q = CV$ to obtain the following equations for energy stored in the capacitor

$$W = U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

1.3 Capacitors in Circuits

1.3.1 Parallel

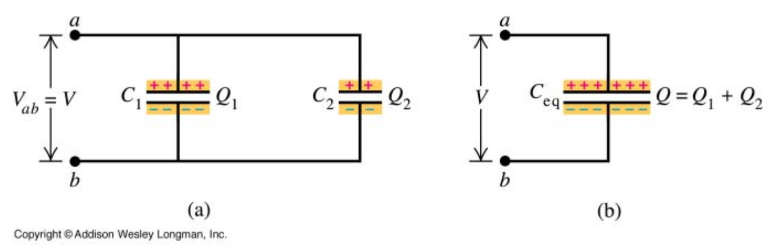

Figure 2: (a) Capacitors in parallel with battery supplying V (b) the equivalent capacitance of the two in parallel.

A capacitor can be charged by connecting the plates to the terminals of a battery, which are maintained at a potential difference V called the terminal voltage. The connection results in sharing the charges between the terminals and the plates. For example, the plate that is connected to the (positive) negative terminal will acquire some (positive) negative charge. The sharing causes a momentary reduction of charges on the terminals, and a decrease in the terminal voltage. Chemical reactions are then triggered to transfer more charge from one terminal to the other to compensate for the loss of charge to the capacitor plates, and maintain the terminal voltage at its initial level. The battery could thus be thought of as a charge pump that brings a charge Q from one plate to the other. The top plates of both capacitors C_1 and C_2 are connected to the positive terminal of the battery and have the same electric potential as the positive terminal. Similarly, both bottom plates are negatively charged and have the same potential as the negative terminal. Thus, the potential difference V is the same across each capacitor. This gives

$$C_1 = \frac{Q_1}{V}$$

$$C_2 = \frac{Q_2}{V}$$

These two capacitors can be replaced by a single equivalent capacitor with a total charge Qsupplied by the battery. However, since Q is shared by the two capacitors, we must have by charge conservation

$$Q = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2)V \longrightarrow \frac{Q}{V} = C_{eq} = C_1 + C_2$$

However, this would work for any number of capacitors in parallel which gives the following equation

$$C_{eq.parallel} = C_1 + C_2 + C_3... + C_N = \sum_{i=1}^{N} C_i \tag{1}$$

1.3.2 Series

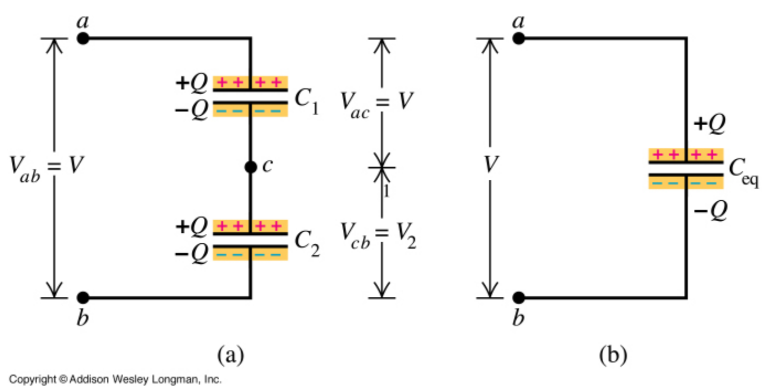

Figure 3: (a) Capacitors in serial with battery supplying V (b) the equivalent capacitance of the two in series. Let $V_{ac} = V_1$ and $V_{cb} = V_2$.

Suppose two initially uncharged capacitors C_1 and C_2 are connected in series, as shown in the figure above A potential difference V is then applied across both capacitors. The top plate of capacitor 1 is connected to the positive terminal of the battery and becomes positively charged with a charge $+Q$, while the bottom plate of capacitor 2 is connected to the negative terminal and becomes negatively charged with charge Q as electrons flow in. What about the inner plates? They were initially uncharged; now the outside plates each attract an equal and opposite charge. So the right plate of capacitor 1 will acquire a charge Q and the top plate of capacitor $+Q$. The potential differences across capacitors C_1 and C_2 are

$$V_1 = \frac{Q}{C_1}$$

$$V_2 = \frac{Q}{C_2}$$

From the figure above, we see that the total potential difference is simply the sum of the two individual potential differences:

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_1} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Dividing out by Q we get the equivalent capacitance in the form of

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

But this can also be generalized to any amount of capacitors in series as follows

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_N} = \sum_{i=1}^{N} \frac{1}{C_i} \tag{2}$$

1.4 Dielectrics

Dielectrics decrease the overall electric field in the capacitor by having an induced an electric field inside the dielectric.

2 Kirchoffs Voltage Loop Law(KVL) and Current Law(KCL)

Kirchoffs Kirchoff's Loop Law states that the closed loop integral of the electric field in a circuit must be zero. Where the line integral of electric field is the electric potential

$$\oint_C \vec{E} \cdot d\vec{\ell} = \sum_{i=1}^{N} V_i = 0$$

Since around a circuit there is a finite amount of circuit elements there is a finite and discrete amount of voltage drops, as seen in the previous equation. This is a very powerful tool. The steps in any multi-loop circuit (including one loop) are as follows

1. Identify a node (particularly one of unknown voltage)and label the currents. You must have at least one current going into the node and at least one current leaving the node. This will one more equation to add to your system of equations.

2. Next, identify the maximum number of independent loops in the network.

3. Label the current around each loop, with known currents where possible (due to current sources), and otherwise with unknown variables. It is important to understand what is meant by loop current. The current through any element is given by the algebraic sum of loop currents of the loops that that element is part of. As such, the loop current is simply a bookkeeping convenience that will be useful for the method. Generally, this process will lead to a unique (but perhaps unknown) current associated with each loop. Furthermore, Kirchoff's current law will be satisfied automatically at each node.

4. For each loop with unknown current, apply Kirchoff's Voltage Law. This will lead to an equation in the unknown loop current. (The equation will also involve other loop currents, namely, for other loops which share a common element.) There is no need to apply KVL for loops with known current. Indeed, such loops include current sources, and the constitutive relation of current sources gives no information about the voltage across the source; hence, it adds no new information that would allow one to find the unknown loop currents.

5. The resulting linear equations are solved for the unknown loop currents.

6. If the voltage across the various network elements are desired, the constitutive relations are used to derive them. In the case of current sources, KVL must be applied to the loop that includes the source to find the source voltage.

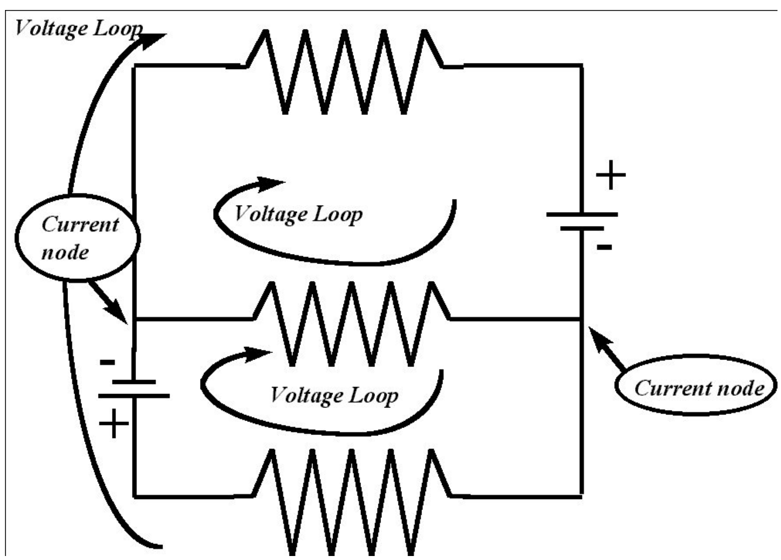

Figure 4: Kirchoffs loop law and current law

3 RC Circuits

Using the relation, $Q = CV$, and knowing the definition of current we can differentiate both sides with respect to time and obtain $\frac{dQ}{dt} = C\frac{dV}{dt}$. Where current as a function of time is defined as $I(t) = \frac{dQ}{dt}$. We can find what the current passing "through" a capacitor is, if there is a changing voltage across the capacitor. This is the main idea of RC circuits.

3.1 RC Circuit with Battery(Charging)

We first consider the case where we have an RC circuit with a switch and a battery, with the switch initially in the open position. When we close the switch current will begin to flow out of the battery and begin building up on each plate of the capacitor. This gradual build up of charge produces a gradual build up of voltage and hence a "current" flowing through the capacitor.

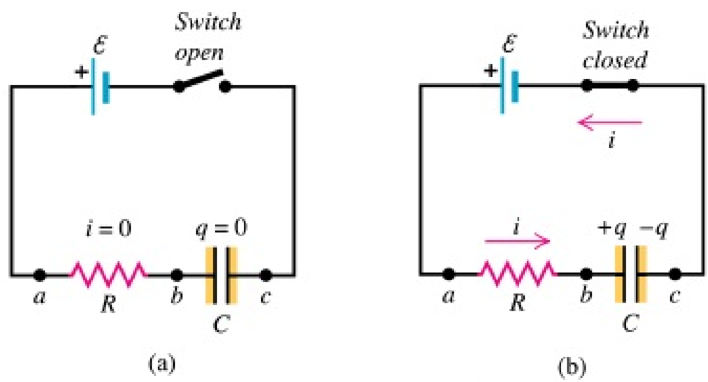

Figure 5: (a) The switch is open and no current flows, hence no charge on capacitor and voltage across it. (b) The switch is closed and current begins to flow, charge builds up on capacitor and voltage across capacitor increases.

By applying Kirchoffs voltage loop law we obtain the following

$$\mathcal{E} - IR - \frac{Q}{C} = 0 \longrightarrow \mathcal{E} - \frac{dQ}{dt}R - \frac{Q}{C} = 0$$

Which is a first order differential equation that is easily solved and unimportant, so I will omit the steps. The solution for charge on the capacitor as a function of time is then

$$Q_C(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{\tau}}\right) \tag{3}$$

Where τ is the time constant defined as $\tau = RC$. The voltage across the capacitor is readily found by using the relation $Q = CV$ with the previous equation

$$V_C(t) = \mathcal{E}\left(1 - e^{-\frac{t}{\tau}}\right) \tag{4}$$

Now to find the current through the capacitor as a function of time all we do is differentiate equation 3 since the definition of current is $I = \frac{dQ}{dt}$

$$\frac{dQ_C(t)}{dt} = I_C(t) = C\mathcal{E} \cdot \frac{-1}{RC}\left(-e^{-\frac{t}{\tau}}\right)$$

Which after simplifying leaves us with

$$I_C(t) = \frac{\mathcal{E}}{R}e^{-\frac{t}{\tau}} \tag{5}$$

Which is an exponentially decreasing function. So after a certain amount of time there will be essentially no current flowing through the capacitor, hence it is like an open switch.

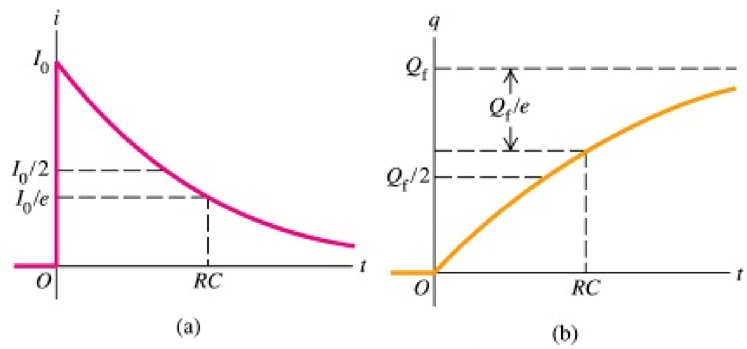

Figure 6: (a) shows how current flows as a function of time. It starts at a maximum and then decreases exponentially. (b) Shows how charge on the capacitor starts at zero then slowly builds up to a maximum value. NOTE: That voltage as a function of time is the same thing as charge just scaled by the capacitor value, seen easily by the relation $Q = CV$

3.2 RC Circuit without Battery (Discharging)

After a capacitor has been charged it then has the potential to do work, or in other words act like a battery itself! We can find the behavior of this type of circuit using the same technique as before. First find the differential equation using Kirchoff's Loop Law then solve. I will omit these steps for it is repetitive. The solutions are then just

$$Q_C(t) = CV_0 e^{-\frac{t}{\tau}} \tag{6}$$

Where the V_0 is usually the $\mathcal{E}$ value of the battery used to charge up the capacitor. We see that the charge starts at a maximum then exponentially decreases towards 0. Differentiating this result we obtain the current

$$I_C(t) = \frac{V_0}{R} e^{-\frac{t}{\tau}} \tag{7}$$

To obtain the voltage as a function of time all we do is scale the current by the capacitor value which gives us

$$V_C(t) = V_0 e^{-\frac{t}{\tau}} \tag{8}$$

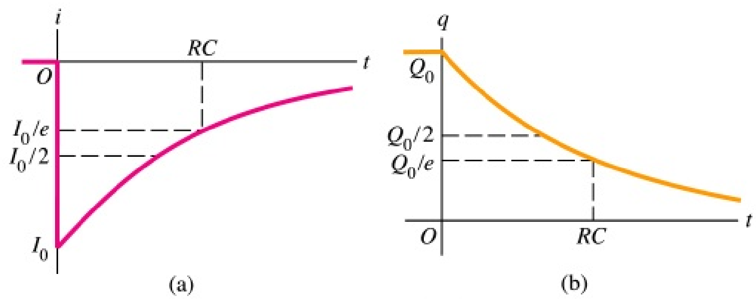

Figure 7: (a) shows how current flows as a function of time. It starts at a maximum (in the opposite direction as before!) and then decreases(in magnitude!) exponentially to zero. (b) Shows how charge on the capacitor starts at maximum then exponentially decreases to a minimum value. NOTE: That voltage as a function of time is the same thing as charge just scaled by the capacitor value, seen easily by the relation $Q = CV$

4 Magnetism

Magnetic field can be defined in many equivalent ways based on the effects it has on its environment. For instance, a particle having an electric charge, q, and moving in a magnetic field with a velocity, $\vec{v}$, experiences a force, $\vec{F}_m$, called the Lorentz force. See force on a charged particle below. Alternatively, the magnetic field can be defined in terms of the torque it produces on a magnetic dipole.

4.1 Magnetic Force on Moving Charged Particle

The equation of the force acting upon a moving charged particle in a magnetic field is given by

$$\vec{F} = q\vec{v} \times \vec{B} \tag{9}$$

$$q\vec{v} \times \vec{B} = q \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \vec{v}_x & \vec{v}_y & \vec{v}_z \\ \vec{B}_x & \vec{B}_y & \vec{B}_z \end{vmatrix} = q \left[\hat{i}(\vec{v}_y\vec{B}_z - \vec{B}_y\vec{v}_z) - \hat{j}(\vec{v}_x\vec{B}_z - \vec{B}_x\vec{v}_z) + \hat{z}(\vec{v}_x\vec{B}_y - \vec{B}_x\vec{v}_y) \right].$$

The implications of this expression include:

1. The force is perpendicular to both the velocity $\vec{v}$ of the charge q and the magnetic field $\vec{B}$.

2. The magnitude of the force is $|\vec{F}| = q|\vec{v}||\vec{B}|\sin(\theta)$ where θ is the angle $< 180°$ between the velocity and the magnetic field. This implies that the magnetic force on a stationary charge or a charge moving parallel to the magnetic field is zero.

3. The direction of the force is given by the right hand rule. The force relationship above is in the form of a vector product.

4.2 Magnetic Force on a Current Carrying Wire

Just as in the case of the charge particle we can say the same thing about a current. Using the definition of current, $I = \frac{\Delta q}{\Delta t}$, and definition of velocity, $\vec{v} = \frac{\Delta \ell}{\Delta t}$ we see that

$$q\vec{v} = q\frac{\Delta \ell}{\Delta t} = \frac{\Delta q}{\Delta t}\ell = I\ell$$

The equation of the force acting upon current carrying wire (charges moving in a wire) in a magnetic field is given by

$$\vec{F} = I\vec{\ell} \times \vec{B} \tag{10}$$

$$I\vec{\ell} \times \vec{B} = I \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \vec{\ell}_x & \vec{\ell}_y & \vec{\ell}_z \\ \vec{B}_x & \vec{B}_y & \vec{B}_z \end{vmatrix} = I\left[\hat{i}(\vec{\ell}_y\vec{B}_z - \vec{B}_y\vec{\ell}_z) - \hat{j}(\vec{\ell}_x\vec{B}_z - \vec{B}_x\vec{\ell}_z) + \hat{z}(\vec{\ell}_x\vec{B}_y - \vec{B}_x\vec{\ell}_y)\right].$$

Where the magnitude of the force is $\mid \vec{F} \mid = I \mid \vec{\ell} \mid \mid \vec{B} \mid \sin(\theta)$ where θ is the angle $< 180°$ between the "wire" vector and the magnetic field. This implies that the magnetic force on a wire without current is zero or a current moving parallel to the magnetic field is zero.

4.3 Right Hand Rule

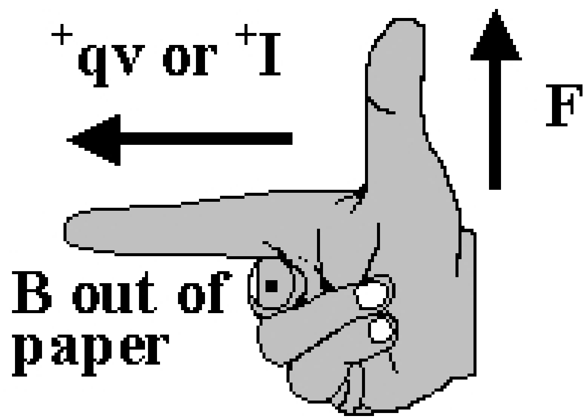

Figure 8: Right hand rule. Note that pointer finger is in direction of either **CONVENTIONAL** current or velocity of **POSITIVELY** charged particle.

When using the Right-Hand Rules, it is important to remember that the rules assume charges move in a conventional current (the hypothetical flow of positive charges). In order to apply either Right-Hand Rule to a moving negative charge, the velocity ($\vec{v}$) of that charge must be reversed–to represent the analogous conventional current. The steps for the right hand rule are as follows

1. Point your index finger in the direction of the charge's velocity, $\vec{v}$, (recall conventional current).
2. Point your middle finger in the direction of the magnetic field, $\vec{B}$.
3. Your thumb now points in the direction of the magnetic force, $\vec{F}_m$.

5 Practice Problems

The following problems are in no way suppose to mimic or even be remotely close to the actual exam. I simply suggest problems that I would put on my own exam.

5.1 Capacitors

5.1.1 Problem 1: 23.41

Find an expression for its capacitance in terms of the dimensions shown below. There is a inner metal rod of radius a and an outer cylindrical shell of radius b. The shell and rod has length ℓ.

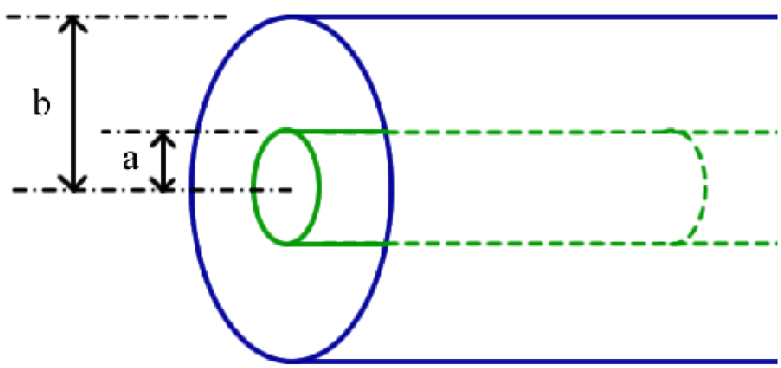

Figure 9: Problem 23.41

Solution:
We know that the capacitance is defined as $C = \epsilon_0 \frac{A}{d}$, but this will not work since we have two different areas. So we use the alternate definition of capacitance, $C = \frac{Q}{V}$, where we put an arbitrary charge on both surfaces. All we need is the electrical potential difference between the inner rod and outer shell when there is some arbitrary charge on the inner rod and outer shell.

By applying Gauss' Law[2] we can find the electric field inside the capacitor ($a < r < b$) to be

$$\vec{E}(r) = \frac{Q}{2\pi\epsilon_0 r\ell}$$

[2]Review Gauss Law if you dont remember how to do this.

Using the definition of electric field, $\vec{E} = -\frac{dV}{d\vec{r}}$, and then integrating with respect to r we obtain the voltage as

$$\int_0^V dV(r) = V = -\int_a^b \frac{Q}{2\pi\epsilon_0 r\ell} = -\frac{Q}{2\pi\epsilon_0\ell}\left[ln\left(r\right)\right]\Big|_a^b = -\frac{Q}{2\pi\epsilon_0\ell}\left[ln\left(b\right) - ln\left(a\right)\right] = \frac{Q}{2\pi\epsilon_0\ell}\left[ln\left(\frac{a}{b}\right)\right]$$

Now we have everything we need to determine the capacitance

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi\epsilon_0\ell}\left[ln\left(\frac{a}{b}\right)\right]} = \frac{2\pi\epsilon_0\ell}{ln\left(\frac{a}{b}\right)}$$

5.1.2 Problem 2: 23.49

A camera flashtube requires $5.0J$ of energy per flash. If the flash lasts $1.0ms$ (a) what power does the flashtube use while it is actually flashing? (b) If the flashtube operates at $200V$ what is the size of the capacitor needed to supply the energy?

Solution:

Part a: Power is defined as work per unit time where work is in Joules. All we need to do here is use that definition.

$$P = \frac{W}{\Delta t} = \frac{5.0J}{1.0 \cdot 10^{-3}s} = 5000W$$

Part b: Using the definition of energy stored in the capacitor we can algebraically solve for C.

$$U = \frac{1}{2}CV^2$$

$$\frac{2U}{V^2} = C$$

Where plugging in the values we get $C = 250/muF$

5.2 Electrical Current

5.2.1 Problem 3: 24.55

A 100% efficient electric motor is lifting a $15N$ weight at $25\frac{cm}{s}$ $(0.25\frac{m}{s})$. If the the motor is connected to a $6.0V$ battery how much current does it draw?

Solution:

Using the definition of both work, $W = \vec{F} \cdot \vec{d}$, and power, $P = \frac{W}{t}$, we obtain

$$P = \frac{W}{t} = \frac{\vec{F} \cdot \vec{d}}{t} = \vec{F} \cdot \frac{\vec{d}}{t} = \vec{F} \cdot \vec{v}$$

Since this motor is 100% efficient the power produced by electrical motor is equivalent to the power used in lifting the weight. Hence we can relate the power found above to electrical power, $P = IV$

$$P = \vec{F} \cdot \vec{v} = IV \longrightarrow I = \frac{\vec{F} \cdot \vec{v}}{V} = \frac{15N \cdot 0.25\frac{m}{s}}{6.0V} = .625A$$

5.3 Circuits

5.3.1 Problem 4: 25.53

For the following diagram find the current in the center vertical resistor, and give its direction.

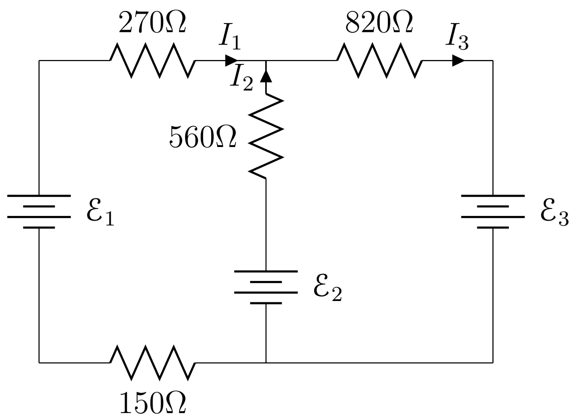

Figure 10: The left loop will be loop 1 and the right loop will be loop 2. The top middle node will be node 1. $\mathcal{E}_1 = 6V$, $\mathcal{E}_2 = 1.5V$, and $\mathcal{E}_3 = 4.5V$

Solution:

Node 1:
As you can see from the diagram above I have labeled the currents I_1, I_2, and I_3 resulting the following form of KCL

$$I_1 + I_2 = I_3 \tag{11}$$

Loop 1:
Using KVL with the loop going in *clockwise* direction we obtain

$$\mathcal{E}_1 - I_1R_1 + I_2R_3 - \mathcal{E}_2 - I_1R_2 = 0 \tag{12}$$

Loop 2:
Using KVL again with the loop going in the *counterclockwise* direction we obtain

$$\mathcal{E}_3 + I_3R_4 + I_2R_3 - \mathcal{E}_2 = 0 \tag{13}$$

Solving the System of Equations: We have 3 equations and 3 unknowns so we can solve for any one of the unknowns. Since we want I_2 we will solve for the other two currents in terms of I_2. From loop 1 (equation 12) we get

$$\begin{aligned}
\mathcal{E}_1 - I_1R_1 + I_2R_3 - \mathcal{E}_2 &= 0 \\
-I_1(R_1 + R_2) &= \mathcal{E}_2 - \mathcal{E}_1 - I_2R_3 \\
I_1(R_1 + R_2) &= \mathcal{E}_1 - \mathcal{E}_2 + I_2R_3 \\
I_1 &= \frac{\mathcal{E}_1 - \mathcal{E}_2 + I_2R_3}{R_1 + R_2} \qquad (14)
\end{aligned}$$

And from loop 2 (equation 13) and using equation 11 we get

$$\begin{aligned}
\mathcal{E}_3 + I_3R_4 + I_2R_3 - \mathcal{E}_2 &= 0 \\
\mathcal{E}_3 + (I_1 + I_2)R_4 + I_2R_3 - \mathcal{E}_2 &= 0 \\
\mathcal{E}_3 + I_1R_4 + I_2R_4 + I_2R_3 - \mathcal{E}_2 &= 0 \\
I_1(R_4) &= \mathcal{E}_2 - \mathcal{E}_3 - I_2(R_4 + R_3) \\
I_1 &= \frac{\mathcal{E}_2 - \mathcal{E}_3 - I_2(R_4 + R_3)}{R_4} \qquad (15)
\end{aligned}$$

No, we have to expressions for I_1 both in terms of known values and the desired I_2. All we do now is set equation 14 equal to equation 15 and solve for I_2

$$\begin{aligned}
\frac{\mathcal{E}_1 - \mathcal{E}_2 + I_2R_3}{R_1 + R_2} &= \frac{\mathcal{E}_2 - \mathcal{E}_3 - I_2(R_4 + R_3)}{R_4} \\
\frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} + \frac{I_2R_3}{R_1 + R_2} &= \frac{\mathcal{E}_2 - \mathcal{E}_3}{R_4} - \frac{I_2(R_4 + R_3)}{R_4 - R_2} \\
\frac{I_2R_3}{R_1 + R_2} + \frac{I_2(R_4 + R_3)}{R_4} &= \frac{\mathcal{E}_2 - \mathcal{E}_3}{R_4 - R_2} - \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} \\
I_2\left(\frac{R_3}{R_1 + R_2} + \frac{(R_4 + R_3)}{R_4}\right) &= \frac{\mathcal{E}_2 - \mathcal{E}_3}{R_4 - R_2} - \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} \\
I_2 &= \frac{\frac{\mathcal{E}_2 - \mathcal{E}_3}{R_4} - \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2}}{\left(\frac{R_3}{R_1 + R_2} + \frac{(R_4 + R_3)}{R_4}\right)} \qquad (16)
\end{aligned}$$

Finally, we can plug in the values for the known circuit elements and find the current I_2. If our answer is negative then we have chosen the wrong direction for the current. You could have plugged in numbers for equations 14 and 15 to obtain simpler equations to use also. The current then is

$$I_2 = -4.74mA$$

This means I chose the direction of the current incorrectly hence the current is flowing downward.

5.4 Magnetism

5.4.1 Problem 6: My Own

An electron is traveling in a straight line in the $+x$ direction at 1% the speed of light. It enters a parallel plate capacitor with an electric field of, $\vec{E} = 500\frac{N}{C}\hat{z}$. There is also a magnetic field, $\vec{B}$, being applied in this capacitor. What is the magnitude and direction of the magnetic field if the electron does not deviate from its straight line trajectory?
Solution:

If the electron does not deviate from its path, then that means the net force acting on the electron is zero, or

$$\sum F = 0$$

Before the electron enters the electric field it has a constant velocity, hence no external forces, but once it enters the capacitor the electric field and magnetic field exert forces on the electronThe force due to the electric field is downward (negative) and the force due to the magnetic field is upward (positive) . Therefore, the sum of the forces on the electron are

$$\sum \vec{F} = -\vec{F}_E + \vec{F}_m = 0 \tag{17}$$

Where $\vec{F}_E = \vec{E}q$ and $\vec{F}_m = q\vec{v} \times \vec{B} = qvBsin(\theta)$. Since the electron is negatively charged it will feel a force in the negative z direction. This means we need a magnetic field that would produce a force on an electron entirely in the positive z direction. By the right hand rule we put our pointer finger in the direction of the velocity of the particle, then our thumb in the positive z direction for the direction of the desired force. Then we curl our index finger so that its normal to the plane our thumb and pointer create (The same as your palm) and that is the direction of the magnetic field. However, this is a negatively charged particle so that it actually feels the force in the opposite direction as we thought!! But don't panic all we need to do is reverse the direction of the original magnetic field. The direction is then just directly out of the page.

The *magnitude* is just found by using the definitions of magnetic and electric force and equation 17

$$\begin{aligned} -\vec{F}_E + \vec{F}_m &= 0 \\ -qE + qvBsin(\theta) &= 0 \\ eE - evBsin(90^\circ) &= 0 \\ vB &= E \\ B &= \frac{E}{v} \end{aligned}$$

The magnitude of the magnetic field is then just $B = 1.666 \cdot 10^{-4}$T

6 References

I used many sources such as Wikipedia, MIT's OCW, SJSU Lecture slides, Giancoli Textbook, etc to make this document as thorough as possible.