Physics 6C Review 1

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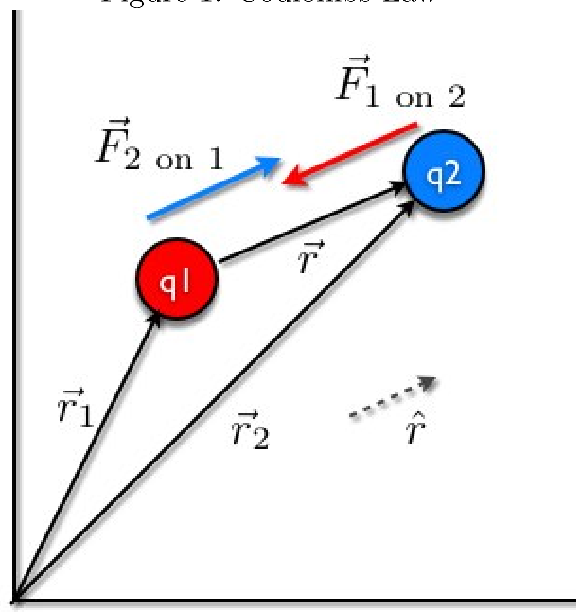
Contents

1 Review

1.1 Coulombs Law

Figure 1: Coulombs Law



The steps for solving any problem of this type are as follows [1]

1. Find $\vec{r}$ by either finding the components of $\vec{r_1}$ and $\vec{r_2}$ and subtracting them from one another. In this case we do $(\vec{r_{1x}}, \vec{r_{1y}}) - (\vec{r_{2x}}, \vec{r_{2y}}) = (\vec{r_x}, \vec{r_y}) = \vec{r}$. We could also use geometry and trigonometry to find the components of the $\vec{r}$.

2. We find $\hat{r}$ by finding the magnitude, $|\ \vec{r}\ |$ and dividing each component of $\vec{r}$ by it. Do as follows, $\hat{r} = \left(\frac{\vec{r_x}}{|\vec{r}|}, \frac{\vec{r_y}}{|\vec{r}|}\right)$. To check to make sure $\hat{r}$ is correct you can square each component and add them together and the result should be equal to one. Or you could use trigonometry [2].

3. We next find the magnitude of the force between the two charges by using Coulomb's Law, $F = \frac{q_1 \cdot q_2}{4\pi\epsilon_0 \cdot r^2}$.

4. Finally, we multiply each component of the unit vector, $\hat{r}$ by the magnitude F. The result should be the components of the force exerted on the charge in question. You can find angle of $\vec{F}$ by using inverse tangent of the y-component over the x-component.

[1] This is not the only way though! I.e. you could also find $\vec{E}$ at the the position of the charge and use $\vec{F} = \vec{E} \cdot q$

[2] Ask me in class about this if you are curious

1.2 Finding Electric Field: Coulombs Way

1.2.1 Description of Electric Field

An electric field is an invisible entity[3] which exists in the region around a charged particle. It is caused to exist by the charged particle. The effect of an electric field is to exert a force on any charged particle (other than the charged particle causing the electric field to exist) that finds itself at a point in space at which the electric field exists. The electric field at an empty point in space is the force-per-charge-of-would-be-victim at that empty point in space. The charged particle that is causing the electric field to exist is called a source charge. The electric field exists in the region around the source charge whether or not there is a victim charged particle for the electric field to exert a force upon. To find the electric we field we would place a tiny charged particle "victim" at the point we are concerned with, and then find the force exerted on our victim. Then we would divide by the victim charge so that our electric field is dependent solely on the source charge. Hence, we would get an answer of units $\frac{N}{C}$, or Force per unit charge.

$$\vec{E} = \frac{\vec{F}}{q_{victim}}$$

By dimensional analysis you can convince yourself that electric field has units of $\frac{V}{m}$ also, or the rate of change of the potential.

1.3 Electric Dipole

Dipoles are common in molecules and dielectric (insulating) materials. All the formulas we were given in class or in the homework about the dipole can be derived from first principles. For example, we can calculate the electric field at any point using Coulomb's law for two point charges. There is a key thing to note that many people forget (including me) is that the electric dipole moment vector points in the opposite direction as the electric field. That is the dipole vector points from negative charge to positive charge. Please refer to Professor Steinacker's excellent description of dipole moments. These are lecture notes 4 and 5.

2 Gauss' Law

2.1 Electric flux

The electric flux,ϕ, is defined as the amount of electric field passing through a given surface area. We can define the flux mathematically as the scalar product between the electric field vector, $\vec{E}$, and the area vector, $\vec{A}$. Please note that the area vector is always normal to the surface! It is written as such,

$$\phi = \vec{E} \cdot \vec{A} = E \cdot A \cdot cos\left(\theta\right)$$

where θ is the angle between the $\vec{E}$ and the $\vec{A}$.

[3]English rather than physics: An entity is something that exists. I use the word entity here rather than thing or substance because either of these words would imply that we are talking about matter. The electric field is not matter.

2.2 Gauss' Law

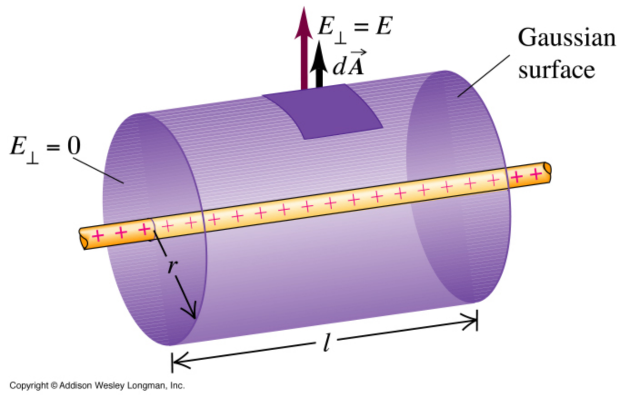
Gauss' Law mathematically is the relation between a surface integral and a volume integral. For physics is the relation between the electric flux,ϕ, and the total charge enclosed by the surface [4] experiencing the flux of the electric field. We want to use symmetry to make the "surface integral" easy, hence

$$\oint \vec{E} \cdot d\vec{A} = \vec{E} \cdot \vec{A}$$

. We do this by choosing a Gaussian surface that has its area vector parallel to the electric field like in the following picture.

2.2.1 Line of Charge

Figure 2: Gauss' Law for Line



The steps to deriving the electric field of a charged rod is as follows.

1. We apply Gauss' Law $\vec{E} \cdot \vec{A} = 4\pi k Q_{enclosed}$, where $A = 2\pi r\ell$ and r and ℓ are the radius and length of the cylinder. Note we do not use the end caps of the cylinder since $\vec{E} \cdot \vec{A} = 0$ for the caps.

2. We then divide by $\vec{A}$ to obtain the answer of

$$\vec{E} = \frac{2kQ_{enclosed}}{r\ell}$$

for total charge known. If the linear charge density is known then its

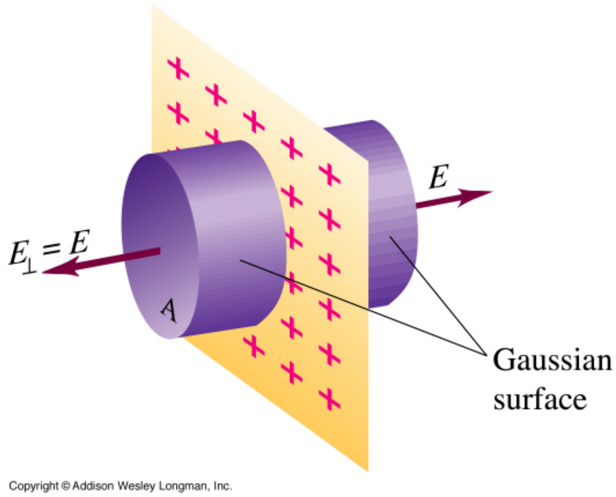
$$\vec{E} = \frac{2k\mu}{r}$$

since $\mu = \frac{Q_{enclosed}}{\ell}$.

[4] This surface is usually called the Gaussian Surface

2.2.2 Gauss' Law for Plane of Charge

Figure 3: Gauss' Law for Plane



Here we apply Gauss' Law again in the same way except $A = 2\pi r^2$ since the electric field is only perpendicular[5] to the Gaussian Surface at the end caps. Hence we have $\vec{E} \cdot \vec{A} = 4\pi k Q_{enclosed}$ where there is some surface charge density σ on the plane. This means the charge enclosed in our Gaussian is $Q_{enclosed} = \sigma \cdot \pi r^2$. We also see that the electric field penetrates both end caps so for the flux we have $\phi = \vec{E} \cdot 2\pi r^2$. Our result is then just

$$\vec{E} = 2\pi k \sigma$$

.

3 Electric Potential

3.1 Definition and Background of Electrical Potential Energy

Potential energy can be defined as the capacity for doing work which arises from position or configuration. In the electrical case, a charge will exert a force on any other charge and potential energy arises from any collection of charges. For example, if a positive charge Q is fixed at some point in space, any other positive charge which is brought close to it will experience a repulsive force and will therefore have potential energy. The potential energy of a test charge q in the vicinity of this source charge will be:

$$U = \frac{KQq}{r}$$

In electricity, it is usually more convenient to use the electric potential energy per unit charge, just called electric potential or voltage.

[5]It is parallel to the area vector!

3.1.1 Zero Potential

The nature of potential is that the zero point is arbitrary; it can be set like the origin of a coordinate system. That is not to say that it is insignificant; once the zero of potential is set, then every value of potential is measured with respect to that zero. Another way of saying it is that it is the change in potential which has physical significance. The zero of electric potential (voltage) is set for convenience, but there is usually some physical or geometric logic to the choice of the zero point. For a single point charge or localized collection of charges, it is logical to set the zero point at infinity. But for an infinite line charge, that is not a logical choice, since the local values of potential would go to infinity.

3.1.2 Potential Reference at Infinity

The general expression for the electric potential as a result of a point charge Q can be obtained by referencing to a zero of potential at infinity. We find the expression for the potential difference by integrating the electric field along a path (any arbitrary path) to obtain:

$$V_b - V_a = kQ\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

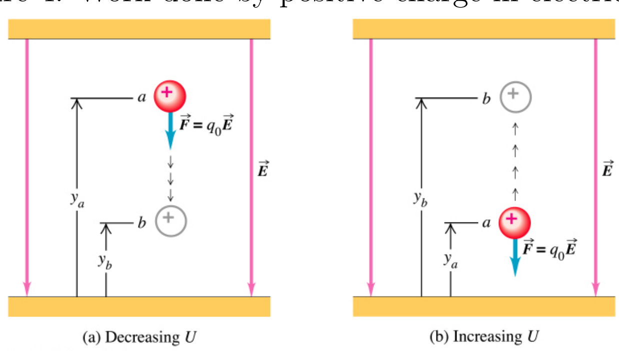
and if we let $r_a \to \infty$ we get just

$$V_b - V_a = \Delta V_{ba} = \frac{kQ}{r_b}$$

. The choice of potential equal to zero at infinity is an arbitrary one, but is logical in this case because the electric field and force approach zero there. Hence, electric potential is determined only by the relative distance between the charge and the point you are concerned with.

3.2 Electric Potential and Work

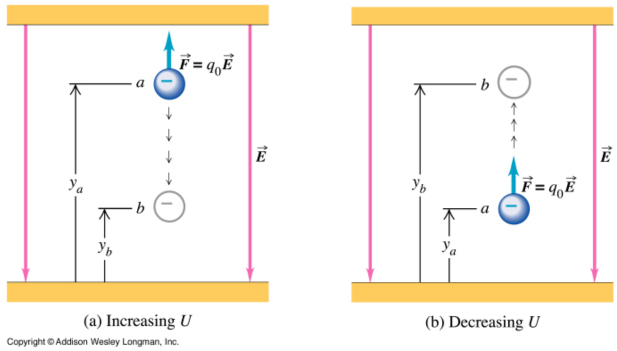
Figure 4: Work done by positive charge in electric field



In the left picture we see that when a positive charge moves in the direction of an electric field, the field does positive work,W, and the potential energy decreases, $W = \vec{F} \cdot \vec{d} = q\vec{E} \cdot \vec{d}$.

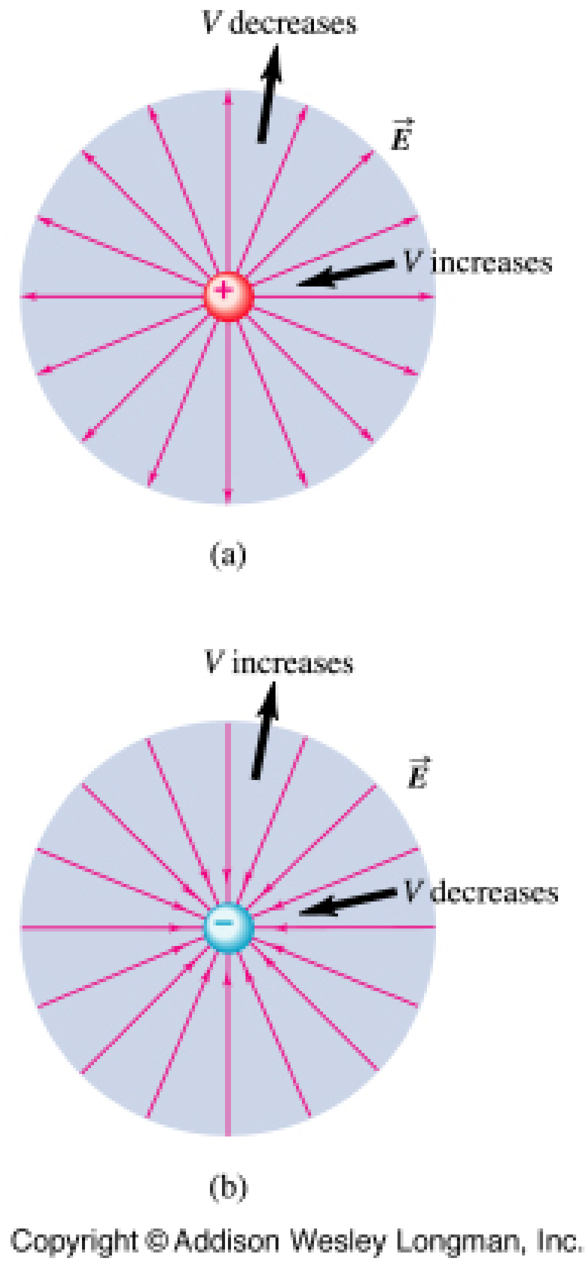
In the right figure, when a positive charge moves in a direction opposite to an electric field, the field does negative work and the potential energy increases.

Figure 5: Work done by negative charge in electric field



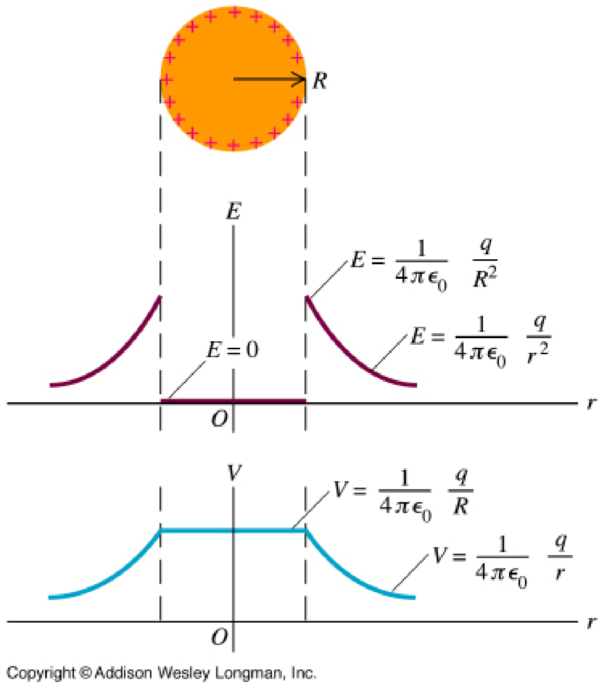
When a NEGATIVE charge moves in the direction of an electric field, the field does negative work and the potential energy increases.

Figure 6: Electric Potential of point charge



In both cases (positive or negative point charge), if you move in the direction of $\vec{E}$, electric potential V decreases; if you move in the direction opposite $\vec{E}$, V increases.

Figure 7: Electric field and potential of solid sphere



The electric field is the negative derivative of the electric potential. Electric field magnitude $\vec{E}$ and potential V at points inside and outside a positively charged spherical conductor.

3.3 Brief Notes of Electric Potential

1. The electric potential energy in a parallel plate capacitor is $V = \vec{E} \cdot \vec{d}$
2. When using energy conservation you must use ΔU NOT ΔV !!
3. The electric field is the negative gradient of the electric potential. Hence,$\vec{E} = -\vec{\nabla} V$. This means that the faster the electric potential changes the larger the electric field.
4. The electric potential energy depends only the final and initial state of the particle (initial is usually taken at $r = \infty \rightarrow V_{initial} = 0$)[6].
5. Work is defined as the change in potential energy, hence $W = \frac{\Delta U_{ab}}{q} = \vec{F} \cdot \vec{d}$

[6]Only for conservative fields! Which for this class is always