Physics 5B Practice Problems

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January 24, 2013

1 Multiple Choice Section

1. The graph shows a plot of displacement versus time for a simple harmonic oscillator. At the time indicated by the solid dot, the velocity and acceleration of the oscillator are

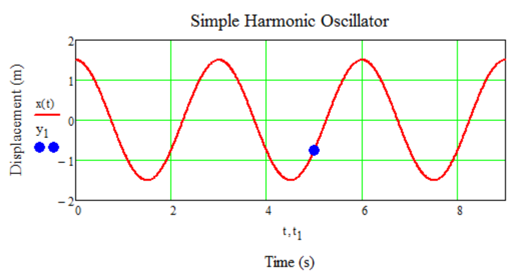

Figure 1: This figure was made with MathCAD and is courtesy of Professor Robert Johnson.

A. $v > 0$ **and** $a > 0$

B. $v < 0$ and $a > 0$

C. $v > 0$ and $a < 0$

D. $v < 0$ and $a < 0$

2. A mass m is hanging from the ceiling of an elevator by a spring of spring constant k. How will acceleration of the elevator affect the frequency of the mass as compared to when the elevator is at rest?

A. The frequency will increase during upward acceleration, and decrease during downward acceleration.

B. The frequency will decrease during upward acceleration, and increase during downward acceleration.

C. The frequency is unaffected by any acceleration.

D. The frequency will decrease for any acceleration.

E. The frequency will increase for any acceleration.

3. Two identical simple harmonic oscillators are set into motion by stretching them from equilibrium and then releasing them from rest. If oscillator A is stretched from equilibrium twice as far as oscillator B, then the maximum velocity of oscillator A will be

A. $\frac{1}{4}$ that of oscillator B.

B. $\frac{1}{2}$ that of oscillator B.

C. equal to that of oscillator B.

D. twice that of oscillator B**.**

E. four times that of oscillator B.

4. Two identical masses are attached to identical springs. They are sitting at equilibrium when suddenly they are given an initial velocity. If Mass 1 has twice the initial velocity as Mass 2, then how will its time to return to equilibrium compare with that of Mass 1?

   A. Mass 2 will take longer

   B. Mass 2 will take less time

   C. Mass 2 will take equal amount of time as mass 1.

5. A grandfather clock, which uses a pendulum for timing, is set to the correct time at noon. But when midnight arrives the clock reads 11:55 (i.e. it lost 5 minutes). To correct the timing, one should

   A. add some mass to the end of the pendulum arm.

   B. increase the length of the pendulum arm.

   C. decrease the length of the pendulum arm.

   D. move to a city with a higher elevation.

6. The magnitude of acceleration of an oscillator is maximum when the oscillator is

   A. at maximum velocity

   B. at equilibrium position

   C. when the position is at half its maximum position

   D. at maximum amplitude

7. The displacement from equilibrium of a harmonic oscillator (0.50 kg mass on a spring) is given by (in MKS units)

$$x(t) = 0.20sin(120 \cdot t + \pi/6)$$

   a) At what minimum $t > 0$ is the mass found to be at equilibrium position of the oscillator? b) What are the speed v and acceleration a of the mass at that time? c) How many times per second will the mass pass through the equilibrium position? d) What is the total mechanical energy E of the oscillator? e) What is the potential energy U of the oscillator at $t = 0$? f) Write an expression for the acceleration a(t) of the mass as a function of time.

8. In a SHO when is the acceleration AND velocity *simultaneously* equal to zero? A is maximum amplitude.

   A. $x = A$

   B. $x = 0$

   C. $x = \frac{A}{2}$

   D. $x = \frac{A}{\sqrt{2}}$

   E. never

9. A mass m is attached to a spring of spring constant k and is oscillating about the equilbrium position at frequency ω. If we double the mass the total energy will ______ and the maximum potential energy will ______. Fill in the blanks respectively.

   A. Remain the same, decrease

   B. Remain the same, increase

   C. Remain the same, remain the same

   D. Increase, decrease

   E. Increase, increase

   F. Increase, remain the same

   G. Decrease, decrease

   H. Decrease, increase

10. A mass m is attached to a string of length ℓ and is oscillating about the equilibrium position at frequency ω. If we double the mass the total energy will ______ and the maximum potential energy will ______. Fill in the blanks respectively.

   A. Remain the same, decrease

   B. Remain the same, increase

   C. Remain the same, remain the same

   D. Increase, decrease

   E. Increase, increase

   F. Increase, remain the same

   G. Decrease, decrease

   H. Decrease, increase

11. A damped SHO of mass m is attached to a spring of spring constant k and is oscillating about the equilibrium position at frequency ω and amplitude A. The mass comes to rest at the equilibrium after 60 periods. What is the total displacement?

   A. 60A

   B. 240A

   C. $\frac{A}{2}$

   D. A

   E. 2A

12. A 300kg wrecking ball hangs from a thick steel wire. When it hits a wall it causes a wave pulse to travel up the wire. How does the speed of the wave on the wire change as it propagates upward?

   A. Decreases

   B. Increases

   C. Stays constant

13. Does a transverse wave or a longitudinal wave travel faster?

   A. Longitudinal

   B. Transverse

   C. Not enough information given.

14. A thick rope is attached to a smaller cord. Wave pulses of equal amplitude, A, are sent towards each other from opposite ends. Assuming that the wave from the smaller cord reaches the wave from the thick rope on the **thick ropes side**, what will the amplitude of the resulting wave be?

   A. Greater than $2A$

   B. $2A$

   C. In between A and $2A$

   D. A

   E. Less than A.

15. A thick rope is attached to a smaller cord. Wave pulses of equal amplitude, A, are sent towards each other from opposite ends. Assuming that the wave from the smaller cord reaches the wave from the thick rope on the **smaller cords side**, what will the amplitude of the resulting wave be?

   A. Greater than $2A$

   B. $2A$

   C. In between A and $2A$

   D. A

   E. Less than A.

2 Free Response Section

16. The displacement from equilibrium of a harmonic oscillator (0.50 kg mass on a spring) is given by (in MKS units)

$$x(t) = 0.20 sin(]120 \cdot t + \pi/6)$$

Solution: The position is zero when the argument of sine is $n\pi$ where n is any integer. Since we want minimum we set $n = 1$. Hence,

$$\begin{aligned} 120t + \pi 6 =& \pi \\ 120t =& 5\pi/6 \\ t =& \frac{5\pi/6}{120} \approx 0.022s \end{aligned}$$

b) What are the speed v and acceleration a of the mass at that time?

Solution: At equilibrium $a = 0$ and $v = v_{max} = \omega A = 120 \cdot 0.20 = 24m/s$

c) How many times per second will the mass pass through the equilibrium position?

Solution: The mass will pass through twice each period. This implies

$$2f = 2\frac{\omega}{2\pi} = \frac{\omega}{\pi} = \frac{120}{\pi} = 38s^{-1}$$

d) What is the total mechanical energy E of the oscillator?

Solution: $E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.5kg)(24m/s)^2 = 144J \approx 140J$

e) What is the potential energy U of the oscillator at $t = 0$?

Solution: The spring constant is found from the angular frequency relation

$$k = \omega^2 m = 120^2 \cdot 0.5kg = 7200N/m$$

Now that we have the spring constant we just use the elastic energy equation to determine the potential energy stored in the spring of the oscillator at $t = 0$.

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(7200)(0.2sin(\pi/6))^2 = 36J$$

f) Write an expression for the acceleration a(t) of the mass as a function of time.

Solution: $a = \frac{d^2x}{dt^2} = -\frac{k}{m}x(t) = -\omega^2 Asin(\omega t + \phi) = -2880sin(120t + \pi/6)m/s^2$

17. A 950-kg car traveling at $25\frac{m}{s}$ hits a spring that compresses 5 meters. What is the spring constant? How long is the car in contact with the spring before the car bounces off in opposite direction?

Solution:
Part A:
The car has only kinetic energy before it hits the spring and it has only potential energy when the spring is fully compressed. Hence, we use energy conservation

$$\begin{aligned} E_i &= E_f \\ \frac{mv^2}{2} &= \frac{kx^2}{2} \\ mv^2 &= kx^2 \\ \frac{mv^2}{x^2} &= k \\ \frac{950kg \cdot (25\frac{m}{s})^2}{(5m)^2} &= k = 23,750N/m \end{aligned}$$

Part B:
Since the collision is elastic (no energy is lost) we can just find the time it takes the spring to stop the car and then multiply that time by two. First we find the acceleration the car feels due to the spring.

$$\begin{aligned} v^2 - v_0^2 &= 2a\Delta x \\ -v_0^2 &= 2a\Delta x \\ \frac{-v_0^2}{2\Delta x} &= a \\ \frac{-(25\frac{m}{s})^2}{2 \cdot 5m} &= a = -62.5\frac{m}{s^2} \end{aligned}$$

Now we just integrate our (average) acceleration with respect to time.

$$\begin{aligned} \frac{dv}{dt} &= a \\ dv &= adt \\ \int_{v_0}^{v} dv &= \int_0^t adt \\ v - v_0 &= at \\ \frac{-v_0}{a} &= t \\ \frac{-25\frac{m}{s}}{-62.5\frac{m}{s^2}} &= t = 0.4s \end{aligned}$$

Therefore the total time the car was in contact with the spring was twice that time,

$$t_{total} = 2t = 0.8s$$

18. A $(m =)0.4kg$ cord is stretched between two supports $(\ell =)7.8m$ apart. When one support is hit with a hammer it sends a transverse wave along the cord. If the wave reaches the other support in $(t =)0.85$ seconds, what is the tension?

Solution: First we must figure out what is the linear mass density of the rope. This is easy, all we have to do is divide the total mass by the total length.

$$\mu = \frac{m}{\ell} = \frac{0.4kg}{7.8m} \approx 0.05\frac{kg}{m}$$

Now, all we do is rearrange the velocity equation to isolate tension, F_T.

$$\begin{aligned} v =& \sqrt{\frac{F_T}{\mu}} \\ v^2 =& \frac{F_T}{\mu} \\ v^2\mu =& F_T \\ \left(\frac{\ell}{t}\right)^2 \frac{m}{\ell} =& F_T \\ \left(\frac{7.8m}{0.85s}\right)^2 \frac{0.4kg}{7.8m} =& F_T = 4.31N \end{aligned}$$

3 Challenge Problem

19. A particle moving under a conservative force oscillates between x_1 and x_2. Show that the period of oscillation is

$$\tau = 2\int_{x_1}^{x_2}\sqrt{\frac{m}{2(V(x_2)-V(x_1))}}dx$$

In particular, if $V = \frac{1}{2}m\omega_0^2(x^2 - bx^4)$, show that the period for oscillations of amplitude a is

$$\tau = \frac{2}{\omega}\int_{-a}^{a}\frac{dx}{\sqrt{a^2-x^2}\sqrt{1-b(a^2+x^2)}}$$

Using the binomial theorem to expand in powers of b, and the substitution $x = asin\theta$, show that for small amplitude the period is approximately

$$\tau \approx \frac{2\pi}{\omega_0}(1+\tfrac{3}{4}ba^2)$$

Solution:
Part A:

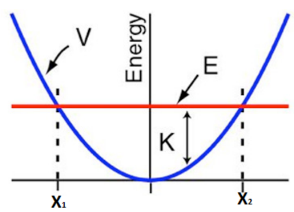

Figure 2: The potential function $V(x) = -m\omega^2\left(\frac{x^2}{2}+\frac{a^4}{2x^2}\right)$

Using energy conservation, and the picture above we see that the velocity of the particle at any point which lies in between $x_1 < x < x_2$ is found by

$$\begin{aligned} T + V =& E = V(x_2) = V(x_1) \\ \frac{mv^2}{2} + V(x) =& V(x_2) \\ m\dot{x}^2 =& 2\left(V(x_2) - V(x)\right) \\ \dot{x} =& \sqrt{\frac{2\left(V(x_2)-V(x)\right)}{m}} \end{aligned}$$

Now that we know the velocity at any point during the oscillation we can solve for the time it takes to get from x_1 to x_2 which is $\frac{1}{2}$ the period. Hence, the period of oscillation ,τ is found by

$$\begin{aligned}
\dot{x} =& \sqrt{\frac{2\left(V(x_2) - V(x)\right)}{m}} \\
\frac{dx}{dt} =& \sqrt{\frac{2\left(V(x_2) - V(x)\right)}{m}} \\
dt =& \frac{dx}{\sqrt{\frac{2(V(x_2) - V(x))}{m}}} \\
2\int dt = \tau =& 2\int_{x_1}^{x_2} \frac{dx}{\sqrt{\frac{2(V(x_2) - V(x))}{m}}} \\
\tau =& 2\int_{x_1}^{x_2} \sqrt{\frac{m}{2\left(V(x_2) - V(x)\right)}} dx
\end{aligned} \tag{1}$$

Part B:
We see that the total energy of oscillator with amplitude a is

$$E = V(a) = \frac{m\omega_0^2}{2}\left(a^2 - ba^4\right)$$

Using the equation we just derived previously for period we get

$$\begin{aligned}
\tau =& 2\int_{x_1}^{x_2} \sqrt{\frac{m}{2\left(V(x_2) - V(x)\right)}} dx \\
\tau =& 2\int_{-a}^{a} \sqrt{\frac{m}{2\left(V(a) - V(x)\right)}} dx \\
\tau =& 2\int_{-a}^{a} \sqrt{\frac{m}{2\left(\frac{m\omega_0^2}{2}\left(a^2 - ba^4\right) - \frac{m\omega_0^2}{2}\left(x^2 - bx^4\right)\right)}} dx \\
\tau =& 2\int_{-a}^{a} \frac{dx}{\sqrt{\left(\omega_0^2\left(a^2 - ba^4\right) - \omega_0^2\left(x^2 - bx^4\right)\right)}} \\
\tau =& \frac{2}{\omega_0}\int_{-a}^{a} \frac{dx}{\sqrt{\left(a^2 - ba^4 - x^2 + bx^4\right)}} \\
\tau =& \frac{2}{\omega_0}\int_{-a}^{a} \frac{dx}{\sqrt{a^2 - x^2}\sqrt{1 - b\left(a^2 + x^2\right)}}
\end{aligned} \tag{2}$$

Part C:
We will use the substitution $x = asin\theta$ and binomial expansion around b. Note $dx = acos\theta d\theta$

$$\begin{aligned}
\tau =& \frac{2}{\omega_0}\int_{-a}^{a}\frac{dx}{\sqrt{a^2-x^2}\sqrt{1-b\left(a^2+x^2\right)}}\\
\tau =& \frac{2}{\omega_0}\int_{-a}^{a}\frac{dx}{\sqrt{a^2-a^2sin^2\theta}\sqrt{1-b\left(a^2+a^2sin^2\theta\right)}}\\
\tau =& \frac{2}{\omega_0}\int_{\theta(-a)}^{\theta(a)}\frac{dx}{acos\theta}\sqrt{1-ba^2\left(1+sin^2\theta\right)}\\
\tau =& \frac{2}{\omega_0}\int_{\theta(-a)}^{\theta(a)}\frac{acos\theta d\theta}{acos\theta}\sqrt{1-ba^2\left(1+sin^2\theta\right)}\\
\tau =& \frac{2}{\omega_0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{1-ba^2\left(1+sin^2\theta\right)}}\\
\tau \approx& \frac{2}{\omega_0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\theta\left(1-\frac{1}{2}ba^2\left(1+sin^2\theta\right)\right)\\
\tau \approx& \frac{2}{\omega_0}\left[\theta-\frac{\theta}{2}ba^2+\left(\frac{\theta}{2}-cos\theta sin\theta\right)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\\
\tau \approx& \frac{2\pi}{\omega_0}\left(1+\frac{3ba^2}{4}\right)
\end{aligned} \tag{3}$$