Physics 6C Review Problems

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August 26, 2012

Contents

1 Problems

1.1 Problem 1: Suspending a Wire with a Current

A horizontal wire carries a current $I_1 = 80A$. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to gravity? The lower wire has a mass of $0.12\frac{g}{m}$.

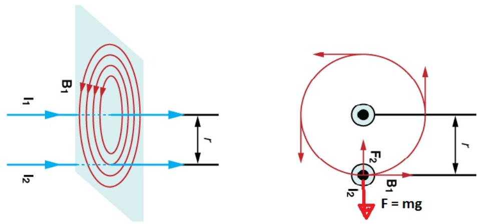

Figure 1: Suspending a current carrying wire with another current carrying wire.

1.2 Problem 2: Force on Rod

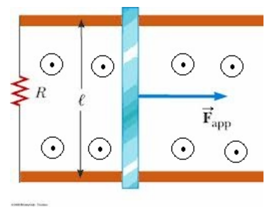

Figure 2: A rod sliding along a conductive track of known resistance in an uniform magnetic field that points out of the paper.

To make the rod of the figure above move to the right at a constant velocity, v, you need to apply a constant force $\vec{F}$ on the rod to the right. (a) What is the force on the rod and (b) what is the power needed to move the rod?

1.3 Problem 3: Generator

The armature of a 60Hz ac generator rotates in a 0.15T, if the area of the coil is $2.0 \times 10^{-2}\text{m}^2$. How many loops should the coil contain if the peak output is $\mathcal{E} = 170\text{V}$?

1.4 Problem 4: LRC Circuit

At $t = 0$, a 40mH inductor is placed in series with a resistance $R = 3.0\Omega$ and a charged capacitor $C = 4.8\mu F$. (a) What is the frequency of oscillation and (b) how long does it take for the charge on the capacitor to drop to half its initial value?

2 Homework Problems

2.1 Extra Credit Problem 28.53

An RLC circuit includes a 1.7H inductor and a 250μ F capacitor rated at 310V . The circuit is connected across a sine-wave generator with $V_\text{p} = 34V$. What minimum resistance will ensure that the capacitor voltage does not exceed its rated value when the circuit is at resonance?

2.1.1 Solution to Extra Credit Problem

First we note that the maximum voltage the capacitor can have is 310V, which means the root-mean-square value must be $V_{\max} = \frac{310V}{\sqrt{2}} = 220V$ [1] secondly, we note that the circuit is in resonance which means the total impedance is a minimum hence

$$\omega L - \frac{1}{\omega C} = 0 \longrightarrow \omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

Where the previous equation comes from the complex impedance form of Ohm's Law of an AC driven LRC circuit

$$I_{RMS} = \frac{V_{RMS_S}}{Z} = \frac{V_{RMS_S}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Where V_{RMS_S} is RMS voltage of the SOURCE, $V_{RMS_S} = \frac{V_p}{\sqrt{2}} = 24V$. Next, we can find the reactance of the capacitor at resonance and subsequently find the RMS current through it at the capacitors maximum RMS voltage.

$$X_C = \frac{1}{\omega_0 C} = \frac{1}{\frac{1}{\sqrt{LC}} C} \longrightarrow X_C = 82\Omega$$

The maximum RMS voltage is 220V so the maximum current flowing through the capacitor must be

$$I_{RMS} = \frac{V_{RMS}}{X_C} = \frac{220V}{82\Omega} = 2.7A$$

Where V_{RMS} is the capacitors voltage. Now, since the maximum current flowing the capacitor is 2.7A is also flowing every other circuit element at the same time we can use the complex impedance form of Ohm's Law AT RESONANCE to determine the minimum resistance to prevent the capacitor from exceeding the maximum voltage.

$$I_{RMS_S} = \frac{V_{RMS_S}}{Z} = \frac{V_{RMS_S}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{24V}{R} \longrightarrow R = \frac{24V}{2.7A} = 8.9\Omega$$

2.2 Problem 28.47

The problem is to determine the inductor needed to be able to pick up the entire FM bandwidth,88MHz to 108MHz. To pick up an FM signal we must be able to create an oscillator at that frequency and observe the resonance. That is how your radio works in essence. We have a variable capacitor that ranges from 14.9pF to 22.9pF. Since the type of oscillator that we will use is an ideal LC circuit the natural frequency that the circuit oscillates at is

$$\omega = \frac{1}{\sqrt{LC}}$$

[1] Even though the max applied voltage is 34V the the capacitor could very well exceed the max applied voltage since the complex nature of circuit analysis. Physically this just means that at some points there could be very large negative voltages across other circuit elements and the capacitor needs to have a large positive voltage to combat the negative voltages.

Which comes from solving the second order differential equation that comes from Kirchoff's Voltage Loop Law. The derivation to this equation is too messy and unnecessary. Just take it on faith. Or google it. We know the frequency range of the given oscillations, the maximum and minimum capacitor values, and that our inductor is a constant; we can match the capacitor values to either the higher frequency or lower frequency. Since the frequency is inversely proportional to capacitance we know that higher frequency, 108MHz, corresponds to the smaller capacitor value, 14.9pF, and that the lower frequency, 88MHz, corresponds to the larger capacitor value, 22.4pF. Now if we take the difference in frequencies we can isolate the inductance value.

$$\begin{aligned}
\omega_H - \omega_L &= \frac{1}{\sqrt{LC_L}} - \frac{1}{\sqrt{LC_H}} \\
2\pi(f_H - f_L) &= \frac{1}{\sqrt{L}}\left(\frac{1}{\sqrt{C_L}} - \frac{1}{\sqrt{C_H}}\right) \\
\sqrt{L} &= \frac{1}{2\pi(f_H - f_L)}\left(\frac{1}{\sqrt{C_L}} - \frac{1}{\sqrt{C_H}}\right) \\
L &= \frac{1}{4\pi^2\left(f_H - f_L\right)^2}\left(\frac{1}{\sqrt{C_L}} - \frac{1}{\sqrt{C_H}}\right)^2 \qquad (1)
\end{aligned}$$

The value of the inductor should be

$$L = 0.14575\mu H$$

2.3 Problem 28.60

2.3.1 Part A

A car battery runs off a $120V_{RMS}$ wall socket. It supplies 10A of current at 14V. This means it produces ($P = 10 \cdot 14V = 140W$) 140W worth of power to the battery. If the charger is 80% efficient at converting power supplied (wall socket) to it (the car battery charger) then the question is how much current needs to be supplied to the charger at $120V_{RMS}$ to supply the desired power to the battery. The power supplied to charger must be 1.25 times larger than the power supplied to the battery.

$$\begin{aligned}
0.8P_{WC} = P_{CB} \longrightarrow P_{WC} &= \frac{5}{4}P_{CB} = P_{WC} \\
\frac{5}{4}P_{CB} &= I_{RMS}V_{RMS} \\
\frac{5P_{CB}}{4V_{RMS}} &= I_{RMS}
\end{aligned}$$

Where P_{WC} is the power supplied from the wall socket to the battery charger, and P_{CB} is power supplied from the battery charger to the battery. Plugging in the numbers that I have it gives a RMS current of

$$I_{RMS} = 1.4583333A \approx 1.5A$$

2.3.2 Part B

It costs $9.5\frac{cents}{kW \cdot hr}$ and the car battery charger uses 175W ($= \frac{140W}{0.8}$) from the wall socket. The cost is then found by dimensional analysis. First we find how many kilowatt hours the battery charger uses.

$$175W \cdot 10hr \cdot \frac{1kW \cdot hr}{1000Whr} = 1.75kW \cdot hr$$

Then multiply by how much one kilowatt hour costs

$$1.75kW \cdot hr \cdot 9.5\frac{cents}{kW \cdot hr} = 16.625cents \approx 17cents$$

3 Solutions to Review Problems

3.1 Solution to Problem 1

We know from Amperes Law that a straight wire produce a circular magnetic field around the wire with a magnitude of

$$B = \frac{\mu_0 I}{2\pi r}$$

We also know that the force on a current carrying wire in a magnetic field is given by

$$\vec{F} = I_u \vec{\ell} \times \vec{B} = I_u \ell B sin(\theta)$$

Combing these two equations we obtain the force on the lower wire due to the upper wire is

$$F = \left(\frac{\mu_0 I_u}{2\pi r}\right) \ell I_l sin(\theta)$$

If we wish to suspend a wire we must make the some of the forces on the lower wire is zero, or the magnetic force plus the force due to gravity equals zero

$$\begin{aligned} \sum F &= 0 \\ F_M - F_W &= 0 \\ F_M &= F_W \\ \left(\frac{\mu_0 I_u}{2\pi r}\right) \ell I_l sin(\theta) &= mg \\ \left(\frac{\mu_0 I_u}{2\pi r}\right) I_l sin(\theta) &= \frac{m}{\ell} g \end{aligned}$$

Where $\frac{m}{\ell}$ is the mass per unit length. Which is what is given to us. We also see from the figure above that the angle between the current and magnetic field is 90°. Now we just solve for I_l

$$I_l = \frac{g}{\left(\frac{\mu_0 I_u}{2\pi r}\right)} \frac{m}{\ell} = 15A$$

3.2 Solution to Problem 2

As we pull the rod in the uniform magnetic field the free electrons in the rod have some velocity, and , hence feel a force given by $F = qvBsin(\theta)$. According to the right hand rule the electrons would move upwards, hence the convential current is downwards in the rod, and clockwise through the loop.

3.2.1 Part A

We know that the rod slides with constant velocity v and hence would move a distance dx in time dt, $dx = vdt$. The induced emf is given by Faraday's Law $\mathcal{E} = -\frac{d\phi_B}{dt}$. For a uniform magnetic field Faraday's Law is just

$$\mathcal{E} = -\frac{\vec{B} \cdot d\vec{A}}{dt}$$
$$\mathcal{E} = -\frac{\vec{B} \cdot \ell v dt}{dt}$$
$$\mathcal{E} = -Bv\ell$$

The induced current is then just $I = \frac{\mathcal{E}}{R}$. Consequently, the force on the rod is

$$F = I\ell B = \frac{\mathcal{E}}{R}\ell B = \frac{B\ell v}{R}\ell B = \frac{B^2\ell^2 v}{R}$$

3.2.2 Part B

The power required to move the rod is the same power that is dissipated by the current flowing since this ideal situation all our force is used to overcome the force on the current that we produce. The power is given by $P = I^2R$ and plugging in values we get

$$P = \left(\frac{B\ell v}{R}\right)^2 R = \frac{(B\ell v)^2}{R}$$

3.3 Solution to Problem 3

We just use our equation for a generator $\mathcal{E} = NBA\omega sin(\omega t)$ however we are only concerned with peak output voltage so we can neglect the oscillatory term. Hence the number of loops needed to produce a 170V output emf is

$$N = \frac{\mathcal{E}}{BA\omega} = \frac{170V}{0.15T 2.0 \times 10^{-2} 60Hz} \approx 945 coils$$

3.4 Solution to problem 4

3.4.1 Part A

Here we just use the equation for frequency of an LRC circuit. The derivation of this frequency is much too in depth and advanced to show here but the result is

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

Plugging in the values we obtain the frequency is

$$\omega = \sqrt{\frac{1}{40mH \cdot 4.8\mu F} - \frac{(3.0\Omega)^2}{4(40mH)^2}} = 360Hz$$

3.4.2 Part B

Using another equation that is unnecessary to derive here

$$Q(t) = Q_0 e^{-\frac{\mathrm{Rt}}{\mathrm{2L}}} cos(\omega t)$$

We can find how long it takes for the charge on the capacitor to discharge to half its original value, or $Q_t = \frac{1}{2} Q_0$.

$$\begin{aligned}
\frac{Q_0}{2} &= Q_0 e^{-\frac{\mathrm{Rt}}{\mathrm{2L}}} \\
\frac{1}{2} &= e^{-\frac{\mathrm{R}}{\mathrm{2L}}\mathrm{t}} \\
ln(\tfrac{1}{2}) &= -\frac{R}{2L} t \\
\frac{2L ln(\frac{1}{2})}{R} &= -t
\end{aligned}$$

Where we disregard the oscillatory envelope ($cos(\omega t)$) since it does not contribute a noticeable decrease in amplitude. The time it takes to reach half is then just

$$t = -\frac{2L ln(\frac{1}{2})}{R} = 18 \times 10^{-3} s$$