Irrational Roots of Quadratic Equations

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Abstract

In this paper I will explore a generalization of the proposition that $\sqrt{2}$ is irrational. In particular, finding roots of a quadratic equation with certain restrictions on the coefficients have only irrational solutions.

1 Theorem

Theorem 1. *If a, b and c are integers satisfying*

- *a is odd,*

- *b and c are even,*

- *c is not a multiple of 4,*

then the equation $ax^2 + bx + c = 0$ has no rational solutions.

2 Proof

2.1 General solution to $ax^2 + bx + c = 0$

We will start with finding a general solution for x. Start with $ax^2 + bx + c = 0$ then complete the square to obtain :

$$(x + \tfrac{b}{2a})^2 = -\tfrac{c}{a} + (\tfrac{b}{2a})^2$$

Then isolate x to get:

$$x = -\tfrac{b}{2a} \pm \sqrt{-\tfrac{c}{a} + \tfrac{b^2}{4a^2}}$$

You may assume all of the standard facts about the arithmetic and order of the rational numbers (including the integers). You may also assume that every rational number may be expressed as the ratio of two integers that have no common factor greater than 1.

2.2 Writing Guidelines

1. You must typeset your paper using LaTeX and the official Math 100 LaTeX-template.

2. If you are having difficulties with LaTeX, then you should seek help in a timely manner. Do not leave these issues to the last minute, and then turn in a half-finished paper.

3. The paper should be between three and four pages in length.

4. Follow all the writing guidelines from your textbook and those discussed in class.

5. Your paper will be graded for the correctness of the mathematics that you present, the clarity and quality of your exposition and the overall structure of the paper. Give careful thought to the order in which the parts of your paper appear.

6. Title your paper appropriately. '*Paper #2 for Math 100, Spring 2012*' is ok, but a title that accurately reflects the mathematical content is better. Don't try to be funny just to be funny.

7. Try hard to eliminate typos and grammatical errors.

8. *Suggestion:* Pair up with another student from the class and proof-read each others' papers.

9. Do not turn in your first or second draft. This paper should be more polished than your first paper.

10. **If it is obvious that no significant effort was put into your paper, you will receive a score that makes no effort to be significantly higher than 0.**

2.3 Collaboration

You are allowed (in fact encouraged) to collaborate on this assignment. However, each student must write his or her own paper. See

http://undergraduate.ucsc.edu/acd_integrity/index.html

for information about what constitutes cheating/plagiarism. For more information, you can read the primer on plagiarism and how to avoid it written by UCSC faculty Gregory S. Gilbert (Environmental Studies) and Ingrid M. Parker (Ecology and Evolutionary Biology), which you can find at

http://scwibles.ucsc.edu/Documents/Avoiding%20Plagiarism.pdf